

Dispersion interactions with long-time tails or beyond local equilibrium

Carsten Henkel

PIERS session 'Casimir effect and heat transfer' (Prahá July 2015)

merci à :

G. Barton (Sussex, UK), B. Budaev (Berkeley, CA)

details in:

'Friction forces on atoms after acceleration'

F. Intravaia & al, *J Phys Cond Matt* **27** (2015) 214020



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Motivation

Dispersion Interactions

Casimir energy

quantum fluctuations

van der Waals force

radiative heat transfer

Planck spectrum

non-equilibrium steady state

nano-scale heating

...

'field theory' vs 'radiation engineering'

Motivation

	fields	matter
approximations	Maxwell	Schrödinger
	$\varepsilon_0 \partial_t \mathbf{E} = \nabla \times \mathbf{H} - \mathbf{j}$
	Coulomb	Newton $\partial_t \mathbf{j} \approx \frac{\rho_0}{m} \mathbf{f}$
	$\varepsilon_0 \nabla^2 \phi = -\rho$	relaxation time approximation $\partial_t f(x, t) = \dots - \frac{f(x, t) - f_{\text{eq}}(x)}{\tau}$ Ohm $\mathbf{j}(\mathbf{r}; \omega) = \sigma(\omega) \mathbf{E}(\mathbf{r}; \omega)$

why?

response of matter system is nonlinear

need approximations: popular is 'Born-Markov'

central assumption: separation of time scales

fast	thermalization
slow	radiative heating

→ local thermal equilibrium $T(\mathbf{r}, t)$

Motivation

Example

Born-Markov master equation for two-level medium (continuous)

$$\begin{aligned}\frac{d\mathbf{P}}{dt} &= -(i\omega_A + \Gamma)\mathbf{P} + i\chi(N_g - N_e)\mathbf{E} \\ \frac{dN_e}{dt} &= -\gamma N_e + \frac{1}{\hbar} \text{Im}(\mathbf{P}^* \cdot \mathbf{E}) \\ \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} &= \nabla \times \mathbf{H} - \frac{d\mathbf{P}}{dt}\end{aligned}$$

separation of time scales:

‘ultra-fast’ response of field, correlation time

Markovian decay

flat (white) spectrum

‘fast’ response of matter, radiative (spontaneous) decay,

‘slow’ re-distribution of populations

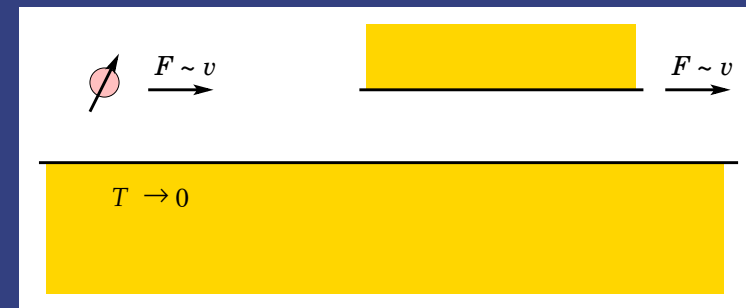
narrow (peaked) spectrum

Outline

Matter response to radiation:
Born-Markov approximation
separation of time scales

This talk:

spectra of (vacuum) field fluctuations
'ultra-slow', 'non-Markovian' correlations



- case study: metallic half-space, two-level system at short distance $z \ll \lambda$

relevant to 'quantum friction':

G. Barton, *Proc Roy Soc (London) A* **453** (1997); *J Phys Cond Matt* **23** (2011)

F. Intravaia & al, *J Phys Cond Matt* **27** (2015)

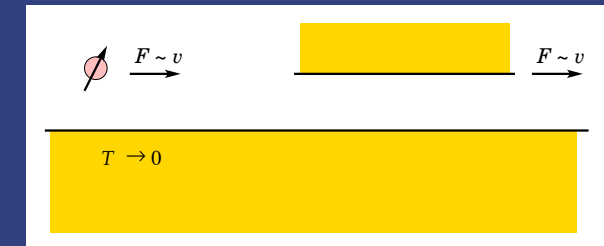
Field response

Barton's model: recap potential

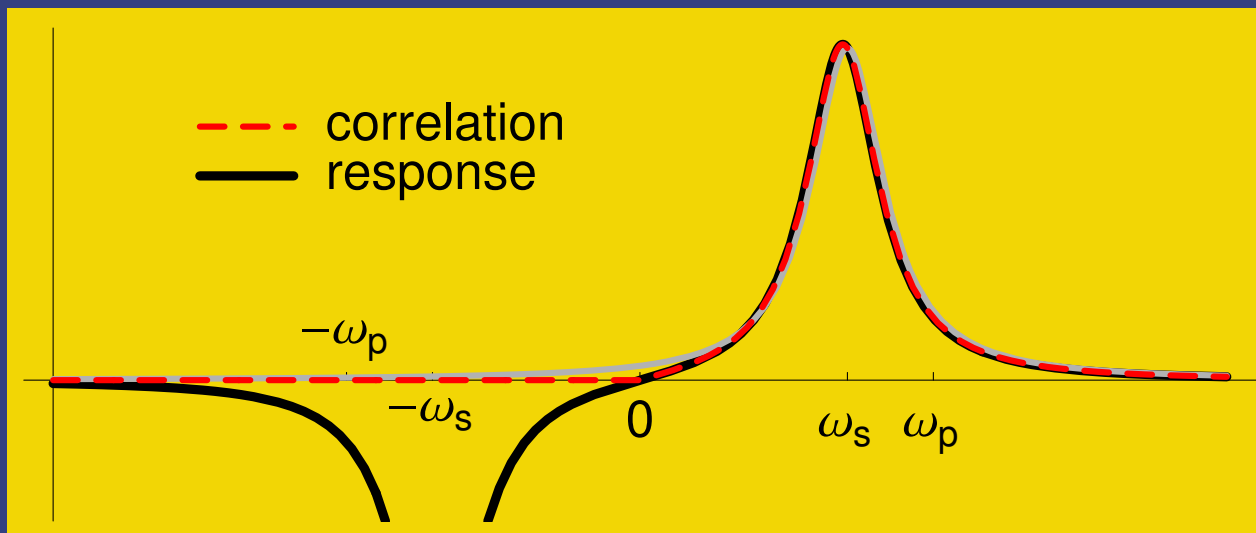
$$\phi(\mathbf{r}, z, t) = \int d^2k \int d\omega \phi_{\mathbf{k}\omega} \exp(i\mathbf{k} \cdot \mathbf{r} - kz) a_{\mathbf{k}\omega}(t) + \text{h.c.}$$

linear response to charge density $\delta\langle\phi(x)\rangle = \int dx' \chi(x, x')\rho(x')$

Kubo formula $\chi(x, x') = -\frac{i}{\hbar} \langle [\phi(x), \phi(x')] \rangle \Theta(t - t')$



$$x = (\mathbf{r}, z, t)$$



Drude metal

surface plasmon $\omega_s = \omega_p / \sqrt{2}$, damping $\Gamma = 0.3 \omega_p$

non-retarded response

$$\chi(z, z'; \mathbf{k}, \omega) = -\frac{e^{-k(z+z')}}{2\epsilon_0 k} R(\omega)$$

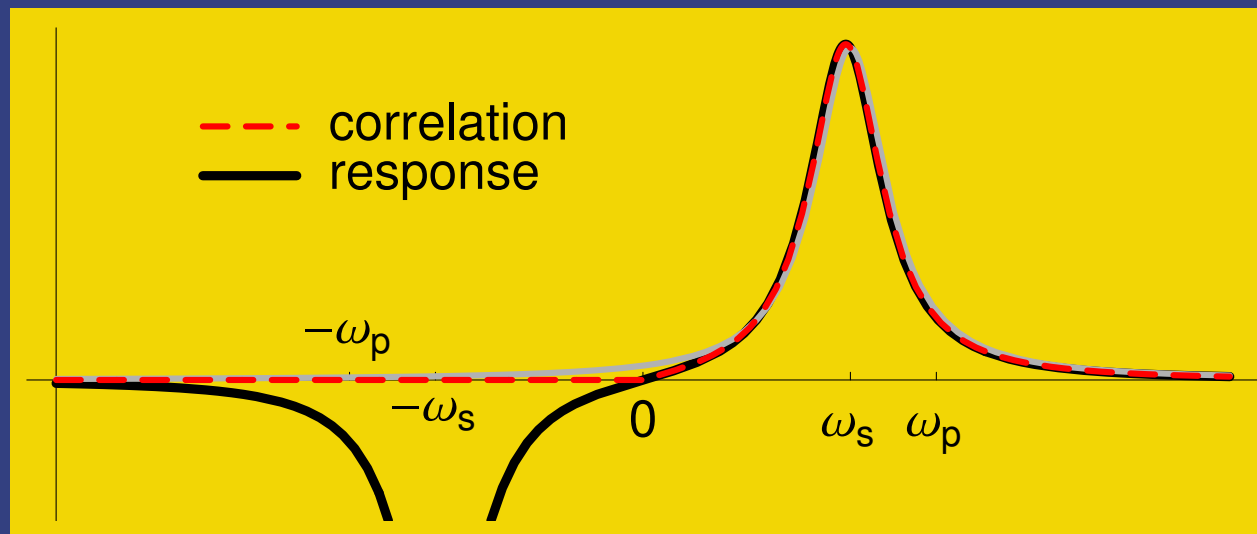
$$R(\omega) = \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 1}$$

Field response

$$\phi(\mathbf{r}, z, t) = \int d^2k \int d\omega \phi_{\mathbf{k}\omega} \exp(i\mathbf{k} \cdot \mathbf{r} - kz) a_{\mathbf{k}\omega}(t) + \text{h.c.}$$

linear response to charge density $\delta\langle\phi(x)\rangle = \int dx' \chi(x, x')\rho(x')$ $x = (\mathbf{r}, z, t)$

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Drude metal

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$$\chi(z, z'; \mathbf{k}, \omega) = -\frac{e^{-k(z+z')}}{2\varepsilon_0 k} R(\omega)$$

back in time domain (damped oscillator)

$$\chi(\vec{r}, \vec{r}', \tau) = -\frac{\omega_s \sin(\omega_s \tau) e^{-\Gamma\tau/2}}{4\pi\varepsilon_0 |\vec{r} - \vec{r}'_{\text{im}}|} \Theta(\tau)$$

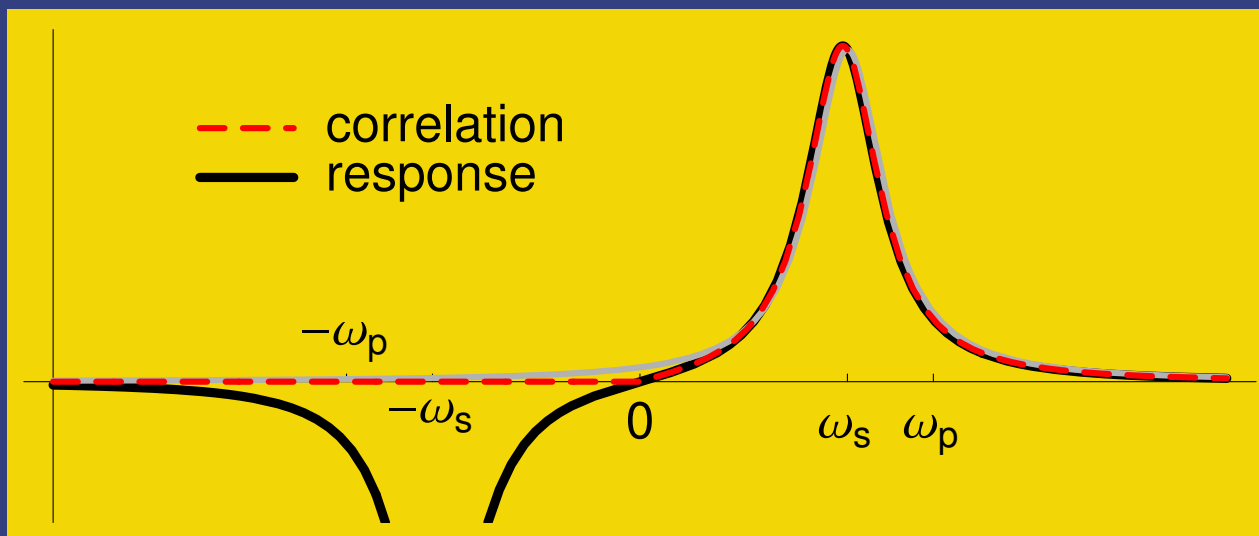
image charge at \vec{r}'_{im}

Field response & correlations

positive frequency part $\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) = \dots a_{\kappa}(t) + \dots a_{\kappa}^{\dagger}(t)$

field vacuum state $\phi^{(+)}(x)|\text{vac}\rangle = 0$

Kubo formula $\chi^{(+)}(x, x') = -\frac{i}{\hbar} \langle [\phi^{(+)}(x), \phi(x')] \rangle \Theta(t - t')$



Drude metal

surface plasmon $\omega_s = \omega_p / \sqrt{2}$, damping $\Gamma = 0.3 \omega_p$

pos freq response

$$\chi^{(+)}(z, z'; \mathbf{k}, \omega) = -i \frac{e^{-k(z+z')}}{\epsilon_0 k} \text{Im } R(\omega) \Theta(\omega) * \frac{1}{i\omega}$$

• algebraic 'fat tail': $\tau \gg 1/\Gamma$

$$\chi^{(+)}(\vec{r}, \vec{r}', \tau) \approx \frac{i \text{Im } R'(0)}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|_{\text{im}}} \frac{\Theta(\tau)}{\pi\tau^2}$$

same tail in correlations $\langle \phi(x)\phi(x') \rangle_{\text{vac}}$
(fluctuation–dissipation / Shiba relation)

Davidson & Kozak, *J Math Phys* **12** (1971);

Wodkiewicz & Eberly, *Ann Phys (NY)* **101** (1976)

Non-Markovian challenges

Comment on fluctuation-dissipation relation (kink near zero frequency)

Shiba relation, Sassetti & Weiss 1990

(Tauber rule) power law tails

...are a problem for Markov approximation ('eternal slip'?)

Haake & Reibold 1985

Challenge: self-consistent field+atom spectral function near zero frequency,
beyond factorising initial conditions

Slutskin & al 2011

Q friction

power law in velocity v depends on shape of spectrum near $\omega = 0$

Intravaia & al 2014/15

Discussing with Bair Budaev

Budaev & Bogy, *Ann Phys (Berlin)* 523 (2011);

Appl Phys Lett 99 (2011)

Thermal radiation with heat current \dot{Q} :

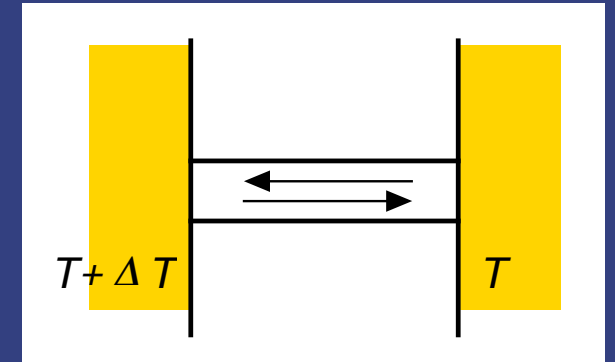
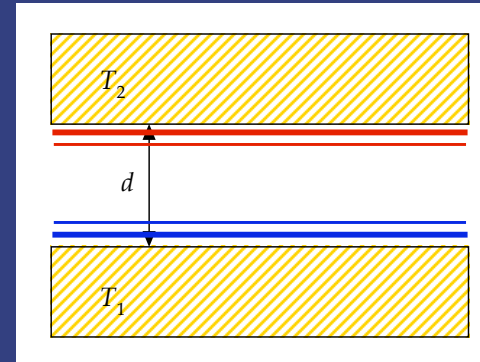
$S(\omega; T, \dot{Q})$ beyond Planck *Ann Phys (Berlin)* 2011

- radiation thermalizes poorly
- matter huge thermal reservoir
- preferred frame: energy current vs crystal lattice

Exit evanescent waves:

reproduce radiative heat transfer without (cit'n?)

- required by boundary conditions (charges, impurities, interfaces, nano-scale objects) *Berry, J mod Opt* 48 (2001)
'virtual photons' tunnelling current
- large $k \gg \omega/c$ – 'non-radiative' transport channels attached to matter near field



signals from astronomy

the Sun: our black body

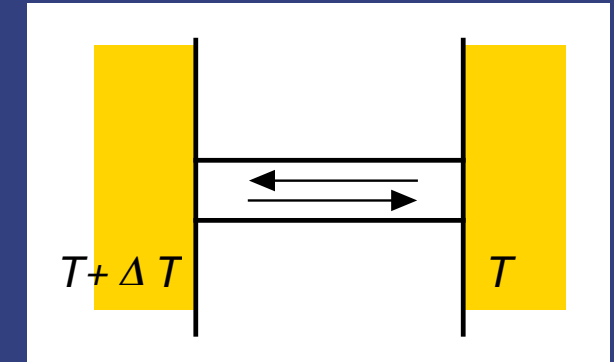
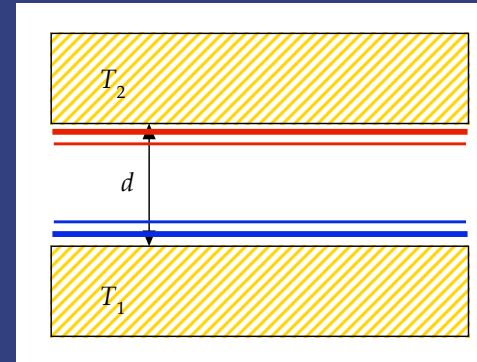
CMB spectroscopy
vs condensed matter

Discussing with Bair Budaev

Thermal radiation with heat current \dot{Q} :
 $S(\omega; T, \dot{Q})$ beyond Planck *Ann Phys (Berlin)* 2011

Exit evanescent waves:
 reproduce radiative heat transfer without (cit'n?)

Short-distance limit of heat transfer:
 recover homogeneous medium



- between conductors $\dot{Q} \sim h(T) \frac{\Delta T}{d^2} \rightarrow \infty$
- between dielectrics $\dot{Q} = \sigma(n)(T_1^4 - T_2^4)$
- radiation model does not allow for $d \rightarrow 0$
- coupled oscillator model ('non-LTE')

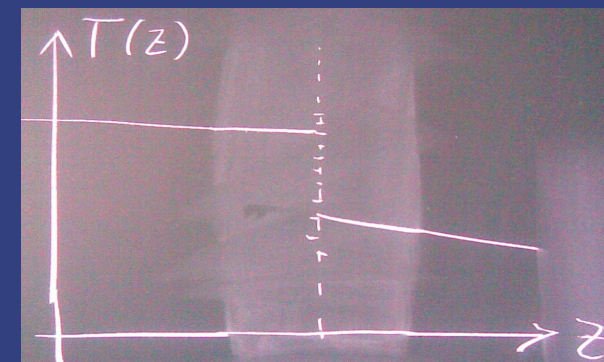
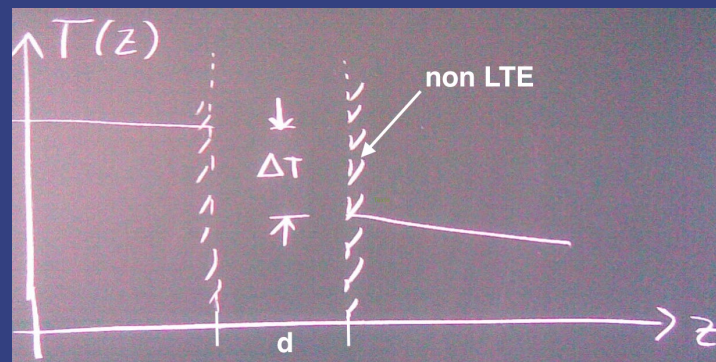
OK! (nonlocality?!)

No! (Kapitza resistance?)

Budaev & Bogy, *Appl Phys Lett* 2011

Ezzahri & Joulain, *Phys Rev B* 2014

e.g. Barton, *J Phys Cond Matt* 27 (2015)



Summary & Perspectives

- Fluctuation-dissipation (Shiba) relation for vacuum field
kink spectrum near zero frequency \rightarrow 'fat correlations'
- Challenge: self-consistent field+atom spectral function near zero frequency
beyond factorising initial conditions
Sasseti & Weiss, *Phys Rev Lett* **65** (1990);
Slutskin & al, *Europhys Lett* **96** (2011)
- 'Easy way out': restore $T > 0$
 $1-10\text{ K} \rightarrow \hbar/k_B T \sim \text{ps}$
- Locally (near) thermal equilibrium
matter dynamics
phonons couple (Chen group, *Nat Commun* 2015)
reasonable approximations behind near-field heat transfer

