On the Foundations of Fluctuation Forces

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G. Pieplow, H. Haakh, J. Schiefele, network 'Casimir' (ESF), DFG

arXiv:1307.0682, *New J Phys* **15** (2013) 023027

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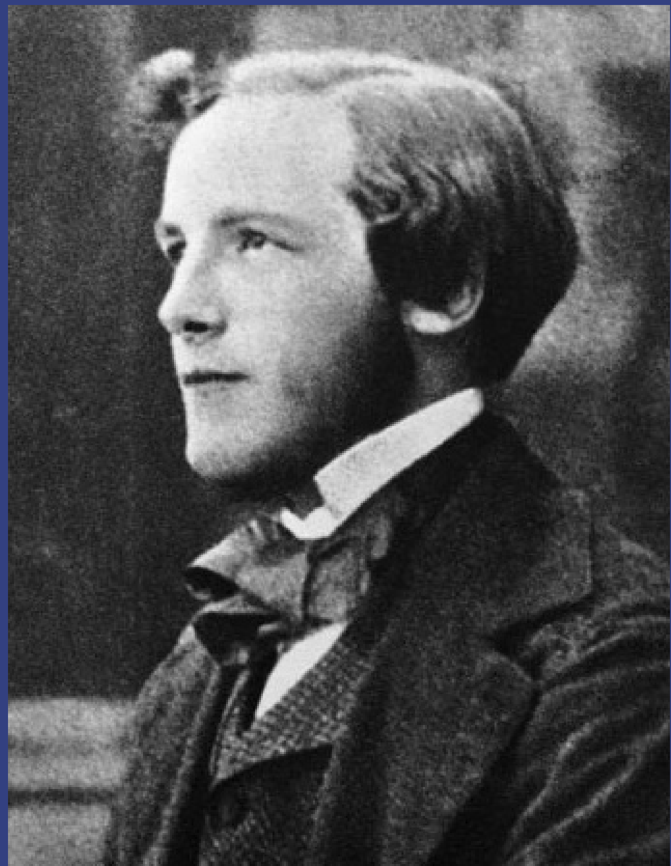
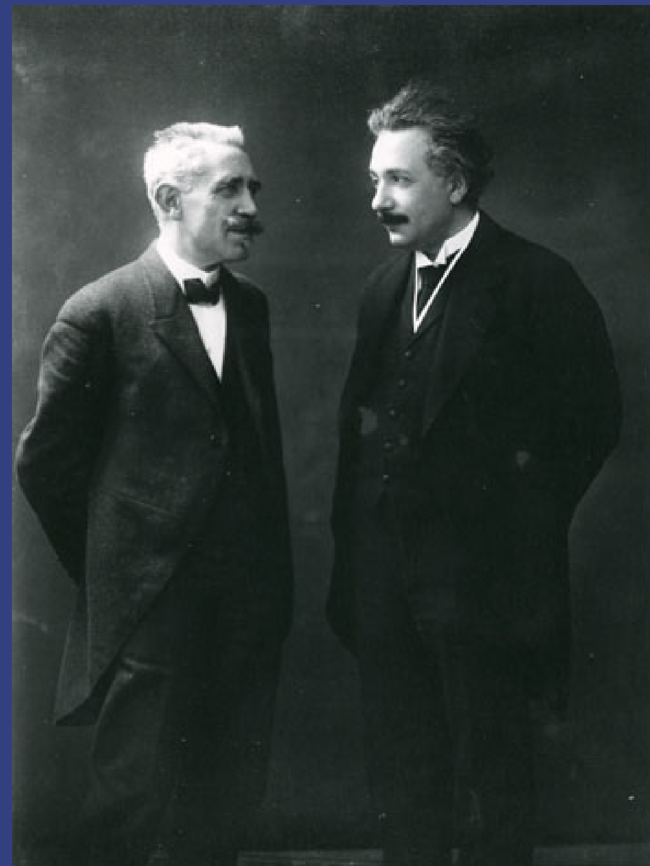
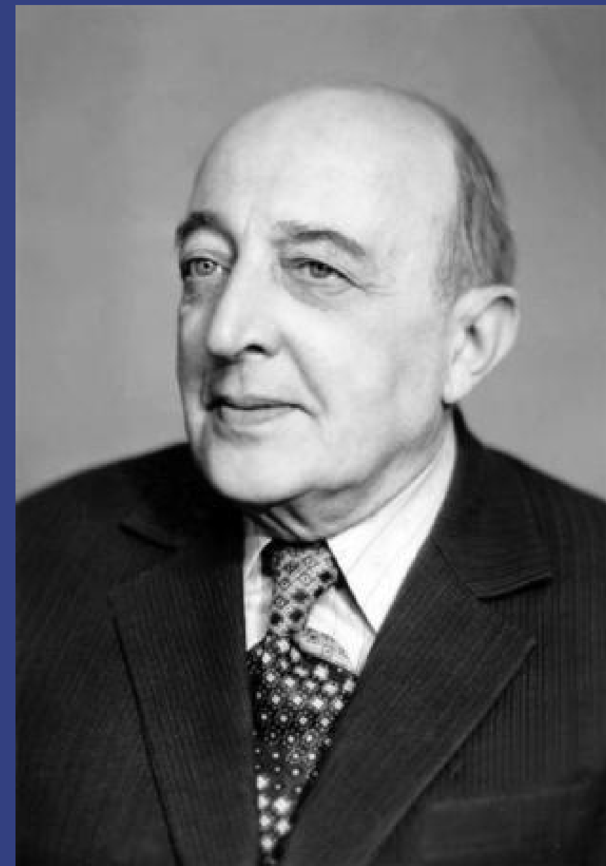


(1831–79)



(1872–1946) (1879–1955)

Paul Langevin



(1908–96)

Sergei Michailovich Rytov

Outline

Why?

Fluctuation interactions

How?

Langevin dialect of Maxwellian = Rytov theory

Basic observables

Local thermodynamic equilibrium

For example?

Non-equilibrium field theory (& Mkrtchian)

Quantum friction (& Pieplow, Haakh, Schiefele)

As time goes by . . .

Potential conversations

Forgotten references

Maxwell & Langevin

Macroscopic electrodynamics

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$
$$\partial_t \mathbf{D} - \nabla \times \mathbf{H} = -\mathbf{j} \qquad \nabla \cdot \mathbf{D} = \rho$$

(inhomogeneous) material

approximation: linear response . . . always matter that provides nonlinearity

$$\mathbf{D} = \varepsilon_0 \varepsilon(\mathbf{x}, \omega) \mathbf{E}$$

$\varepsilon(\omega)$, $\mu(\omega)$ must be complex

$$\mathbf{H} = \mu_0^{-1} \mu^{-1}(\mathbf{x}, \omega) \mathbf{B}$$

$\varepsilon(\mathbf{x})$, $\mu(\mathbf{x})$ cannot be local

[Wed 3P3b, Wed 3P5, Thu 4P4]

Rytov: losses come with fluctuating sources (Langevin forces)

Principles Stat Radiophys (Springer 1989)

Maxwell & Langevin

Macroscopic electrodynamics

$$-\mathrm{i}\frac{\omega}{c^2}\varepsilon(\mathbf{x},\omega)\mathbf{E} = \nabla\times\mu^{-1}(\mathbf{x},\omega)\mathbf{B} - \mu_0\mathbf{j}(\mathbf{x},\omega) \qquad -\mathrm{i}\omega\mathbf{B} = \nabla\times\mathbf{E}$$

Rytov: losses come with fluctuating sources (Langevin forces)

$$\mathbf{j} = \mathbf{j}_{\text{free}} - \mathrm{i}\omega\mathbf{P}(\mathbf{x},\omega) + \nabla\times\mathbf{M}(\mathbf{x},\omega)$$

'noise polarization' $\mathbf{P}$

$$\rho = \rho_{\text{free}} - \nabla\cdot\mathbf{P}(\mathbf{x},\omega)$$

'noise magnetization' $\mathbf{M}$

Maxwell-Langevin equation: 'stochastic differential equation'

$$0 = \langle\mathbf{P}(\mathbf{x},t)\rangle$$

$$0 \neq \langle\mathbf{P}(\mathbf{x},t)\mathbf{P}(\mathbf{x}',t')\rangle = \int\frac{\mathrm{d}\omega}{2\pi}S_P(\mathbf{x},\mathbf{x}',\omega)\,\mathrm{e}^{\mathrm{i}\omega(t-t')}$$

spectral density

Principles Stat Radiophys (Springer 1989)

Maxwell & Langevin

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spectral density

Rytov: $$S_{P,ij}(\mathbf{x},\mathbf{x}',\omega) \approx 2\hbar\bar{N}(\omega)\varepsilon_0\,\mathrm{Im}\,\varepsilon_{ij}(\mathbf{x},\omega)\delta(\mathbf{x}-\mathbf{x}')$$

. . . fluctuation-dissipation relation

Bose-Einstein distribution $\bar{N}(\omega)$, local equilibrium $T \longmapsto T(\mathbf{x})$

Principles Stat Radiophys (Springer 1989)

Fluctuation forces

Field energy (density, in vacuum)

$$u(x) = \frac{\varepsilon_0}{2}\langle \mathbf{E}(x)\cdot\mathbf{E}(x)\rangle + \frac{\mu_0^{-1}}{2}\langle \mathbf{B}(x)\cdot\mathbf{B}(x)\rangle = \int_{-\infty}^{+\infty} \mathrm{d}\omega\, u(x,\omega)$$

- thermal sources $\longmapsto$ blackbody radiation (Planck 1900)
- near objects: non-universal spectrum, distance dependence (Planck, Purcell)

Dorofeyev & Vinogradov (*Phys Rep* 2011)

Force on rigid body in vacuum

$$\frac{\mathrm{d}}{\mathrm{d}t}(\text{total momentum})_i = \oint_{\text{surf}} \mathrm{d}A_j \langle T_{ij}\rangle \qquad \langle\text{Maxwell stress tensor}\rangle$$

$$\langle T_{ij}\rangle = \varepsilon_0\langle E_i E_j - \frac{\delta_{ij}}{2}\mathbf{E}\cdot\mathbf{E}\rangle + \mu_0^{-1}\langle B_i B_j - \frac{\delta_{ij}}{2}\mathbf{B}\cdot\mathbf{B}\rangle$$

- quantum fluctuations $\longmapsto$ Casimir force (cosmological constant?)

$T_A > T_B$ or ∇T: Heat current (Poynting vector) (this session Mon 1P3)

Potential Conversations

— 'Dynamical fluctuations' and 'static' zero-point energy

— Stress tensor in a medium (in/homogeneous) T Philbin, Ch Raabe & D-G Welsch, M v Laue

— EM Fields: (retarded) link between charges R Feynman & J A Wheeler

— Linear macroscopic response: 'filter theory' signal engineers

— Local equilibrium assumption: 'incoherent summation'
coherence from propagation (diffraction) Ch Huyghens, T Young
FD relation valid for both bosonic and fermionic matter
H. B. Callen & T. A. Welton, F. Garcia de Abajo

— Beyond local equilibrium: 'thermodynamic cut' L Boltzmann, M Lax, U Weiss

— Quantum limit ($T_A, T_B \longmapsto 0$):
establishing correlations (entanglement) between bodies J S Høye & I Brevik, I Klich, R Behunin

Example 1: Non-equilibrium field theory

Goal: calculate field correlations with Schwinger-Keldysh technique
diagrammatic formulation of density operator dynamics
path integral evaluation of effective action

Mkrtchian & Henkel
arXiv:1307.0682

Janowicz & Holthaus
Phys Rev A 2003

Sherkunov
Phys Rev A 2007/09

Symmetrized correlations of vector potential ('Keldysh Green function')

$$
\begin{aligned}
D^K(x, x') &= -\mathrm{i}\langle\{A(x), A(x')\}\rangle \\
= \ldots &= \int \mathrm{d}a\, \mathrm{d}a'\, G(x, a) S_J(a, a') G^*(a', x')
\end{aligned}
$$

(retarded) Green function $G(x, x')$, surface current correlations $S_J(a, a')$

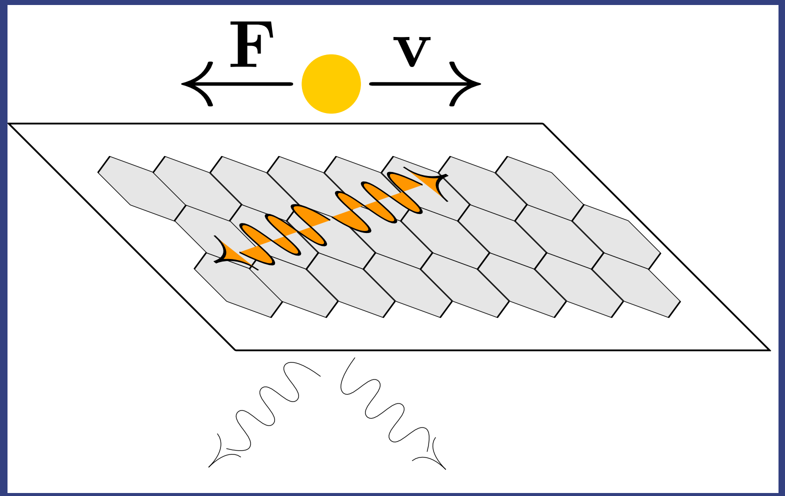
Assumption: un-correlated surfaces (bodies)

$\longrightarrow$ single-interface $S_J(a, a')$ reproduces Rytov theory

+ arbitrary reflection matrices, + Lorentz transformation of surface currents

– planar surfaces (translation symmetry), – stationary situation (spectra)

Example 2: Quantum friction near graphene



metallic nanoparticle at speed $v \sim c$

graphene sheet (dielectric substrate)

- friction ('Coulomb drag')

$\uparrow$ creation of excitations

photons in substrate ("Cherenkov")

graphene plasmons

Mon 1A1, ... Tue 2AK, Wed 3AK

Motivation

giant anomalous Doppler shift: $\omega' = \gamma(\omega - \boldsymbol{k} \cdot \boldsymbol{v}) < 0$

bridge gap between metallic (UV) and graphene (IR) plasmons

controversy on quantum friction ($T \to 0$ limit)

...40 years of discordant results (Teodorovich 1978 ... session Tue 2P4)

Radiation forces and friction

Force on neutral, polarizable particle

$$\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{v}) = \underbrace{\langle d_i \nabla E_i \rangle}_{\text{dominates}} + \langle \mu_i \nabla B_i \rangle$$

↗ fluctuating particle dipole

↘ fluctuating field & substrate excitations

Expansion in 4th order correlations:

— polarizability $\alpha(\omega')$ in co-moving frame: Doppler shift $\omega' = \gamma(\omega - \boldsymbol{k} \cdot \boldsymbol{v})$

— dipole fluctuation spectrum $S_d(\mathbf{x}_A, \omega')$ in local equilibrium

— field response = Green function . . .

Quantum friction force (substrate & field $T_F \to 0$, particle $T_A \to 0$)

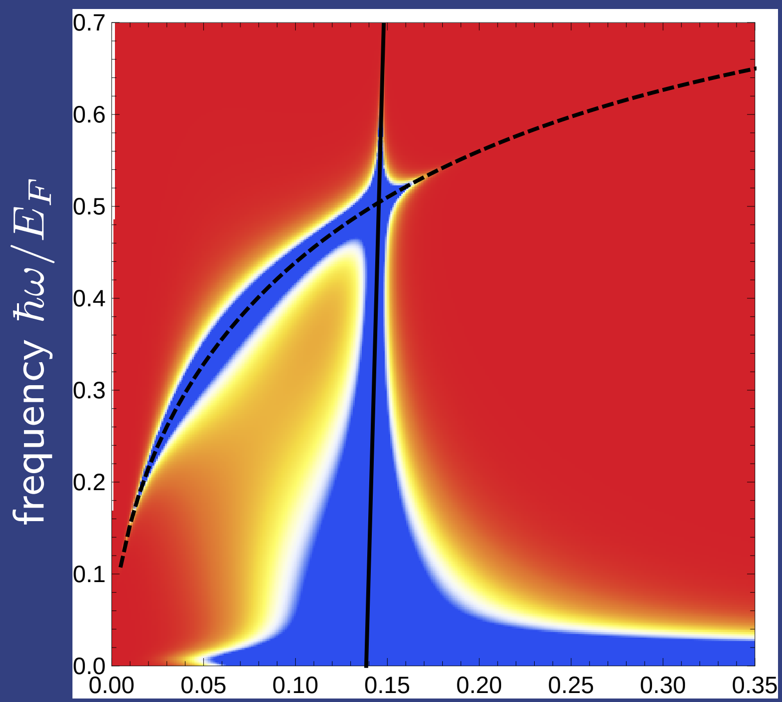
$$F_x = \frac{\hbar}{2\gamma} \int \frac{\mathrm{d}\omega}{2\pi} \frac{\mathrm{d}^2 k}{(2\pi)^2} k_x \left(\mathrm{sgn}\, \omega - \mathrm{sgn}\, \omega' \right)$$

$$\times \operatorname{Im} \alpha(\omega') \sum_{\sigma = s,p} \phi_\sigma(\boldsymbol{k}, \omega) \operatorname{Im} \left(\frac{\mathrm{e}^{-2\kappa z_A} r_\sigma(\boldsymbol{k}, \omega)}{\kappa} \right)$$

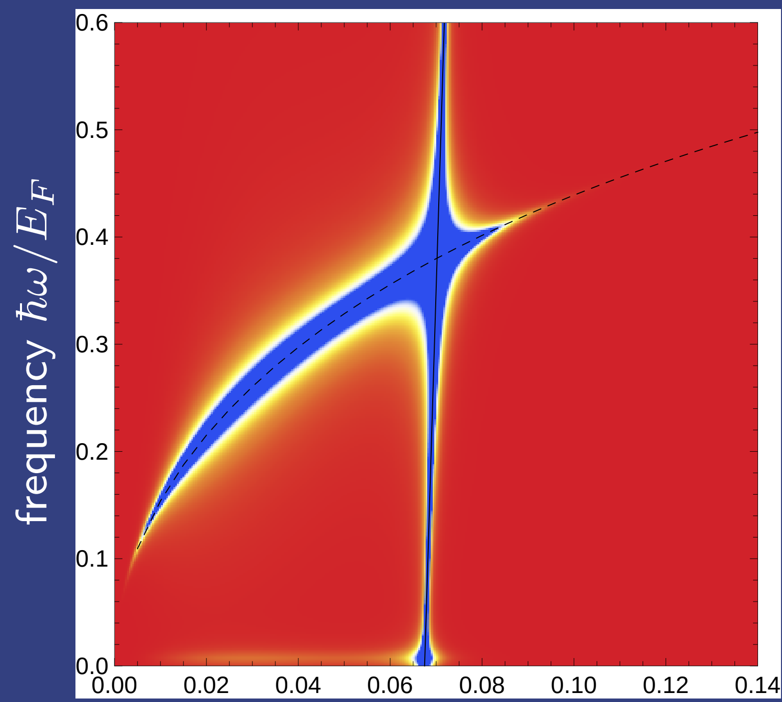
Dedkov & Kyasov, *Phys Solid St* ≥ 2001; Pieplow & Henkel, *New J Phys* 2013

Resonant emission of graphene plasmons

force density $F_x \, \mathrm{d}\omega \, \mathrm{d}k_x$



wave vector k_x/k_F $(v_x = 0.22\,c)$



k_x/k_F $(v_x = 0.42\,c)$

plasmon dispersion

$$\omega \sim \sqrt{k} \ll ck$$

shifted particle resonance

$$-\Omega = \omega' = \gamma(\omega - \boldsymbol{k} \cdot \boldsymbol{v})$$

free-standing graphene film

$E_F = 0.5\,\mathrm{eV}$, $c \sim 300\, v_F$

distance $100\,\mathrm{nm}$

gold particle plasmon $\hbar\Omega \approx 5.2\,\mathrm{eV}$

G. Pieplow, H. Haakh, J. Schiefele, work in progress

giant anomalous Doppler shift: $\omega' < 0$

emission of 1st plasmon excites the particle: $\omega' + \Omega = 0$

Frank & Tamm explain Cherenkov radiation (1937)

review: Ginzburg (*Phys Uspekhi* 1996)

Appendix – forgotten references

von Laue: Thermal radiation in absorbing bodies (*Ann. Phys. (Leipzig)* 1910)
Bakker & Heller: 'Quantum' Brownian motion in electric resistances (*Physica* 1939)
Jauch & Watson: Phenomenological Quantum-Electrodynamics (*Phys Rev* 1948)
Callen & Welton: Irreversibility and generalized noise (*Phys Rev* 1951)
De Groot & al: Series on relativistic thermodynamics (*Physica* $\geq$ 1953)
V. L. Ginzburg: Electrical fluctuations (*Fortschr Phys* 1953)
Linder: Thermal Van der Waals interactions (*J Chem Phys* $\geq$ 1960)
van Kampen: FD relation in non-linear systems? (*Physica* 1960) vs Polevoi & Rytov (*Theor Math Phys* 1975)
Morris & Fürth: Spatial field correlations near conducting surfaces (*Physica* 1960)
see also Fuchs (*Radiophys Quant Electr* 1965)
Jaynes & Cummings: Quantum vs semiclassical radiation theories (*Proc. IEEE* 1963)
Agarwal: FD theorems and series on field quantization (*Z Phys* 1972; *Phys Rev A* 1975)
Boyer: Connection between Rytov and quantum electrodynamics (*Phys Rev D* 1975)
Ginzburg & Ugarov: Macroscopic stress tensor (*Sov Phys Usp* 1976)
Polevoi: Tangential forces/friction in non-equilibrium fields (*JETP* 1985/90)
Eckhart: Problems with FD relations for heat transfer (*Opt Commun* $\geq$ 1982)
Henry & Kazarinov: Quantum noise in photonics (*Rev Mod Phys* 1996)

Summary – Status of Learning Curve

Rytov fluctuation electrodynamics

'Robust working horse'	– as long as matter responds linearly
Universal framework to recover:	thermal radiation, heat transfer,
	Casimir-Lifshitz forces, quantum friction
... beyond local equilibrium:	non-local response

Statistics (thermodynamics) vs quantum

Intuitive picture for vacuum fluctuations	'semiclassical', 'dynamical'
Radiative infinities regularized by	temperature and coupling to matter
... non-equilibrium interactions	entropy production
	entanglement production

www.quantum.physik.uni-potsdam.de → download slides (end of week)