Problem set 1 Hand out: 13 Apr 2011 hand in: 27 Apr 2011

Problem 1.1 – Formatvorlagen von Zeitschriften der Physik (5 points)

Zeitschriften stellen ihren Autoren Formatvorlagen ("templates") zur Verfügung. Teilen Sie sich in Gruppen auf und finden Sie die Vorlagen für die unten angegebenen Zeitschriften. Dokumentieren Sie Ihre Ergebnisse in einer Übersicht, geben Sie an, wer was gemacht hat, und laden Sie ein Dokument, das alle lesen können, auf die moodle-Plattform der Vorlesung (`https://moodle.uni-potsdam.de/course/category.php?id=1774`). Jeder Studierende sucht sich eine Zeitschrift und eine Vorlage aus und füllt diese so weit aus, dass Sie die nächste Übungsaufgabe in dieser Form elektronisch abgeben können.

Nature Physics, Nature Photonics, Europhysics Letters, Optics Letters, Physical Review A, Journal of Physics B, European Physical Journal D, Journal of Optics A, Journal of modern Optics, Optics Communications, Journal of the Optical Society of America B, Applied Physics B, Annalen der Physik (Berlin), Annals of Physics

Problem 1.2 – Observables for a thermal bath (10 points)

In the lecture, we have introduced the "system+bath" Hamiltonian $H_S + H_B + V$ where the pieces relevant to the bath are

$$H_B = \sum_k \hbar\omega_k a_k^\dagger a_k, \tag{1.1}$$

$$V = S^\dagger B + \text{h.c.} = S^\dagger \sum_k \hbar g_k a_k + \text{h.c.} \tag{1.2}$$

with the usual bosonic operators a_k and some system operator S. The "bath spectral density" is defined by

$$J(\omega) = 2\pi \sum_k |g_k|^2 \delta(\omega - \omega_k) \tag{1.3}$$

(1) Give the physical dimensions of B and $J(\omega)$, assuming for simplicity that S is dimensionless. (2) Calculate $B(t)$ in the interaction picture with respect to V and prove the commutator

$$\left[B(t), B^\dagger(t')\right] = \hbar^2 \int \frac{\mathrm{d}\omega}{2\pi} J(\omega)\, \mathrm{e}^{-\mathrm{i}\omega(t-t')} \tag{1.4}$$

What is the frequency domain in this integral? (3) Assume that the bath modes are in thermal equilibrium (with respect to H_B, temperature $T = \hbar/(\beta k_B)$) and calculate the autocorrelation function

$$\langle B(t) B^\dagger(t') \rangle = \hbar^2 \int \frac{\mathrm{d}\omega}{2\pi} J(\omega) \, \frac{\mathrm{e}^{-\mathrm{i}\omega(t-t')}}{1 - \mathrm{e}^{-\beta\omega}} \tag{1.5}$$

(4) Repeat the derivation for the hermitean bath observable $A(t) = B(t) + B^\dagger(t)$. Show that Eqs.(1.4, 1.5) formally still apply, provided the spectral density has the symmetry $J(-\omega) = -J(\omega)$.

Problem 1.3 – An exactly solvable oscillator model (5 points)

Consider the following toy model of an oscillator (coordinate x) that is linearly coupled to another oscillator (coordinate q):

$$H = \frac{1}{2} x^2 + \frac{1}{2M} q^2 - gxq \tag{1.6}$$

(0) Interpret the symbols appearing here and speculate about the units. (1) Identify the bath Hamiltonian H_B and calculate the bath partition function (*Zustandssumme*) Z_B in thermal equilibrium. (2) Calculate the "reduced partition function"

$$Z = \frac{1}{Z_B} \int \mathrm{d}x \, \mathrm{d}q \, \exp(-\beta(H - fx)) \tag{1.7}$$

where f is a control parameter that couples to the system. Compare to the result for the thermalized system,

$$Z_S = \int \mathrm{d}x \, \exp(-\beta(H_S - fx)) \tag{1.8}$$

where H_S is the system Hamiltonian. Which limits can you take to make Z and Z_S "equivalent up to physically irrelevant details" (e.g., a constant factor, a re-interpretation of parameters, . . .)?