Einführung in die Quantenoptik II Sommersemester 2012

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Problem Set 5 Hand out: Wed 29 May 13 hand in: Wed 19 Jun 13

Problem 5.1 – An essay on quasi-probabilities (15 points)

> Geben Sie Ihre Lösung in elektronischer Form ab, unter Benutzung einer beliebigen Formatvorlage (siehe Blatt 1) einer physikalischen Zeitschrift. Achten Sie auf die Ästhetik Ihrer Formeln und Abbildungen. Sprache: Englisch oder Deutsch.

In the lecture, we have seen in several places "phase space plots" for quantum states of a single mode of the radiation field. Write an essay on this technique (two or three pages). You should cover the standard list of quantum states: Fock, Boltzmann, Glauber, and squeezed. The easiest way to calculate phase-space distribution functions is the Q- or Husimi function

$$Q(\alpha) = \mathcal{N}\langle\alpha|\rho|\alpha\rangle \tag{5.1}$$

Find the normalization factor $\mathcal{N}$ (it is independent of the state ρ) such that the integral over the complex plane (with measure $\mathrm{d}^2\alpha = \mathrm{d}\,\mathrm{Re}\,\alpha\,\mathrm{d}\,\mathrm{Im}\,\alpha$) gives unity. Give some details on the calculation of the Q-function for these states and illustrate the results with plots for suitably chosen parameters. [Bonus points:] a discussion why there are alternative phase-space distribution functions (to be found in any textbook on quantum optics).

Problem 5.2 – (5 points)

In the lecture, we have seen the beam splitter transformation. In the real world, transmission and reflection coefficients with any phase and frequency dependence appear. You study here how this can be implemented in quantum optics.

(i) Show using the techniques introduced in the lecture that the unitary transformation

$$S = \exp[\mathrm{i}(z a^\dagger b + z^* b^\dagger a)] \tag{5.2}$$

implements the more general beam splitter matrix

$$S^\dagger \begin{pmatrix} a \\ b \end{pmatrix} S = \begin{pmatrix} c & \mathrm{i}\,s \\ \mathrm{i}\,s^* & c \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{5.3}$$

where c is real, s is complex with $c^2 + |s|^2 = 1$.

(ii) Show that if we want to implement a matrix like in Eq.(5.3) with a frequency dependence, i.e., $c(\omega)$, $s(\omega)$, then the following construction works

$$S = \exp\left[\mathrm{i}\int \mathrm{d}\omega\,(z(\omega)a^\dagger_\omega b_\omega + \mathrm{h.c.})\right] \tag{5.4}$$

with a complex function $z(\omega)$ and where the commutation relations are

$$[a_\omega, a^\dagger_{\omega'}] = \delta(\omega - \omega') \tag{5.5}$$

with a similar relation for b_ω. All other commutators are zero.