Nano-Optik und Plasmonik Sommersemester 2014

Carsten Henkel & Svetlana Santer

Übungsblatt 3

hand out: Wed 07 May 14
hand in: Thu 22 May 14

Problem 3.1 – Photon tunnelling (5 points)

Consider a small gap between two surfaces. In the lower surface (index n_1), a beam is totally reflected and gives rise, in the gap, to an evanescent wave proportional to $t_1 \exp[\mathrm{i}kx - \kappa z]$ where t_1 is the transmission coefficient and k, $\mathrm{i}\kappa$ are the complex components of the wave vector. (The z-axis points from the medium 1 into the gap.) (1) Justify in words why the other surface gives rise to a reflected wave proportional to

$$t_1 r_2 \exp[\mathrm{i}kx + \kappa(z - 2d)] \tag{3.1}$$

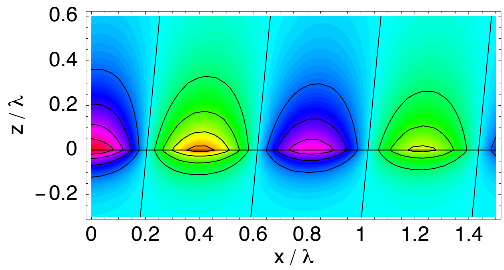
(No need to calculate r_2 – you already know the result, right?) (2) Assume that the sum of these two fields describes the magnetic field component H_y (the other components being zero) and calculate E_x and E_z. (3) Calculate the time-averaged Poynting vector

$$S_z = \mathrm{Re}\,[E_x^* H_y] \tag{3.2}$$

and show that it does not depend on x, z and is proportional to $\mathrm{e}^{-2\kappa d}$ and to the imaginary part of r_2. Why is this so?

Problem 3.2 – Picture of a surface plasmon resonance (5 points)

The following plot is an illustration of a surface plasmon polariton.



Use your knowledge on the continuity of the fields at an interface to make sense out of this plot (taken from *Contemp. Phys.* **48** (2007) 183): (0) what is the wavelength of this plasmon? is the answer non-ambiguous? (1) which medium is metallic, which one dielectric? (2) which electromagnetic field or intensity component is plotted? (3) in which direction does the surface plasmon propagate? (4) is the frequency ω or the wave vector k complex, or both? (5) why are the wave fronts tilted in both media?

Problem 3.3 – Scattering from a small sphere (10 points)

How does an electromagnetic wave scatter off a spherical object? This problem has been solved in 1908 by Gustav Mie, and generations of physicists have evaluated his formulas numerically. The Mie solution is an infinite series over electromagnetic multipoles. Somewhere on the web, you may find the following formulas for the lowest Mie coefficients (electric and magnetic dipoles)

$$a_1 = \frac{A_n(y) - mA_n(x)}{A_n(y) - mB_n(x)} \frac{\psi_n(x)}{\zeta_n(x)}, \qquad (n = 1) \tag{3.3}$$

$$b_1 = \frac{mA_n(y) - A_n(x)}{mA_n(y) - B_n(x)} \frac{\psi_n(x)}{\zeta_n(x)}, \qquad (n = 1) \tag{3.4}$$

where the 'Mie parameter' is $x = \omega a/c$ (a: sphere radius), and $y = \sqrt{\varepsilon}\, x$ ($\varepsilon = m^2$: relative dielectric constant of the sphere). The functions $A_n(x)$ and $B_n(x)$ are logarithmic derivatives of Riccati-Bessel functions

$$A_n(x) = \frac{\psi_n'(x)}{\psi_n(x)}, \quad \psi_n(x) = x j_n(x), \tag{3.5}$$

$$B_n(x) = \frac{\zeta_n'(x)}{\zeta_n(x)}, \quad \zeta_n(x) = x[j_n(x) + \mathrm{i} y_n(x)], \tag{3.6}$$

where the spherical Bessel functions we need here are given by

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}, \quad y_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x} \tag{3.7}$$

(1) Check that the Mie coefficients are dimensionless. (2) For a nano-sphere (small compared to the wavelength), we can expand the Mie coefficients for $x \to 0$, using

$$x \to 0: \quad j_1(x) = \frac{x}{3} + \mathcal{O}(x^3), \quad y_1(x) = -\frac{1}{x^2} + \mathcal{O}(x) \tag{3.8}$$

Compare the corresponding expressions to the Clausius-Mossotti polarizability (here, also $y \to 0$ is taken)

$$\alpha = 4\pi a^3 \frac{\varepsilon - 1}{\varepsilon + 2} \tag{3.9}$$

and to the lowest-order magnetic polarizability (here, y remains arbitrary)

$$\beta = -2\pi a^3 \left[1 + \frac{3}{y}(\cot y - 1/y)\right] \tag{3.10}$$

Find the dimensional factor between the polarisabilities and the Mie coefficients. (3) Make a plot of all the formulas appearing here as a function of frequency in the visible band. Consider spheres of size 1 nm, 10 nm, 100 nm made from dielectric and metallic material. (You can assume that the dielectric function is constant for the dielectric and follows a Drude model for the metal.)