Nano-Optik und Plasmonik

Sommersemester 2014

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Übungsblatt 4

hand out: Wed 18 Jun 14
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Problem 4.1 – Space and time (8 points)

(1) In a numerical method like FDTD, the time step Δt and a spatial grid step like Δx cannot be chosen independently. Find out from information about FDTD methods why the time step must satisfy the inequality

$$\Delta t \le \frac{\Delta x}{c} \tag{4.1}$$

How is this criterion modified in a medium with a complex refractive index $n + \mathrm{i}k$ like a metal?

(2) For a metal described by the Drude model, the response of the medium (dielectric function) can be represented in terms of a current density $\mathbf{j}$. Show from the Drude dielectric function that Ohm's law (conductivity σ) takes the following form in the time domain

$$\partial_t \mathbf{j}(\mathbf{x}, t) = -\frac{\mathbf{j}(\mathbf{x}, t)}{\tau} + \frac{\sigma}{\tau}\mathbf{E}(\mathbf{x}, t) \tag{4.2}$$

and formulate an FDTD version of this equation. What would be your guess for the ratio $\Delta t/\tau$?

Problem 4.2 – Rayleigh's dynamic theory of gratings (12 points)

'On the dynamical theory of gratings', Lord Rayleigh, *Proc. R. Soc. London A* **79** (1907) 399–416

Consider a relief grating with a surface $z = h(x)$ which is periodic with period a. A scalar field $\phi(x, z)$ is incident on this grating and satisfies the wave equation

$$(\triangle + k^2)\phi = 0\,, \qquad z \ge h(x) \tag{4.3}$$

as well as the 'hard wall boundary condition':

$$\phi(x, h(x)) = 0 \tag{4.4}$$

At large distances $z \to \infty$, the field is the sum of an incident field and a reflected (scattered) field

$$z \to \infty: \qquad \phi(x, z) = \phi_{\text{in}}\, \mathrm{e}^{\mathrm{i}k(x\sin\theta - z\cos\theta)} + \phi_{\text{refl}}(x, z) \tag{4.5}$$

(1) Explain why the reflected field can be written in the form

$$\phi_{\text{refl}}(x,z) = \sum_n a_n \, \mathrm{e}^{\mathrm{i}(k\sin\theta + nq)x + \mathrm{i}k_{zn}z}\,, \qquad (k\sin\theta + nq)^2 + k_{zn}^2 = k^2 \tag{4.6}$$

with $\operatorname{Im} k_{zn} \geq 0$ for large n. Calculate for the nth diffracted wave $\phi_n(x,z) = a_n \, \mathrm{e}^{\mathrm{i}(k\sin\theta + nq)x + \mathrm{i}k_{zn}z}$ the 'Poynting vector' (current density)

$$\mathbf{j}_n = \operatorname{Im}\left(\phi_n^* \nabla \phi_n\right) \tag{4.7}$$

and discuss the cases of real and imaginary k_{zn}.

(2) Make the assumption (Rayleigh approximation) that the asymptotic form (4.5) is valid down to the surface $z = h(x)$ and calculate a_n for a corrugation small compared to the wavelength: $k|h(x)| \ll 1$.

(3) In the Kirchhoff approximation, one assumes that the grating surface reflects like a planar surface would, except for an additional phase shift. This yields a reflected field

$$\phi_{\text{refl}}(x,z) = -\mathrm{e}^{-2\mathrm{i}kh(x)\cos\theta} \, \mathrm{e}^{\mathrm{i}k(x\sin\theta + z\cos\theta)} \tag{4.8}$$

Consider a relief $h(x) = h\cos qx$ and calculate the diffraction amplitudes a_n.