Einführung in die Quantenoptik II

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Hinweis. Die Übungsaufgaben sind ein Versuch, verschiedene 'Geschmäcker' zu bedienen: mal geht es um Abschätzungen, Einheiten, Größenordnungen. Mal gibt es einiges zu rechnen. Häufig ist schon das Interpretieren des Aufgabentextes Teil der Herausforderung.

Es gilt die Regel: Lassen Sie sich von Fehlern in den angegebenen Formeln nicht verwirren. Im Zweifelsfall fehlt eben im Aufgabentext ein Faktor 2, π, i, -1 ...

Problem 1.1 – Laser feeling (10 points)

Please look up the typical elements of a laser: a cavity for the laser mode, an active medium, a mechanism for pumping the medium, an optical system to shape the laser beam.

Look up typical numbers for a laser:

(1) the product ('quality factor' Q) between the resonance frequency ω_c and the 'photon lifetime' τ in the cavity. In the lecture, we shall use $\kappa = 1/\tau$ as 'cavity decay rate'.

(2) the ratio between the input power (needed for pumping the active medium) and the optical output power. This 'efficiency' η is often not very large – this is one of the reasons why laser-induced fusion is probably not a very efficient way of producing energy, for example.

(3) the product of cavity lifetime τ and the flux of photons (photons per second) in the laser beam. We shall see that this is a good estimate for the photon number $\langle n \rangle$ in the cavity. Try to find an estimate. For a 'microlaser' or 'micro maser' (experiments by Nobel prize winner Serge Haroche in Paris), this number is small.

(4) try to find from a 'laser catalogue' information about the fluctuations (stability) of the laser beam power. What physical quantities are used to describe this? We shall be interested in the lecture in the standard deviation Δn of the photon number: think how one could translate the catalogue information into the number Δn.

Problem 1.2 – Handling field modes (10 points)

In electrodynamics, you may have learned that the density of modes of the electromagnetic field in free space is

$$\rho_\omega(\omega) = \frac{\omega^2}{\pi^2 c^3} \tag{1.1}$$

(1) Check the dimension (unit) of this quantity and find out in which sense this is a density of modes. Switch to wavelength $\lambda = 2\pi c/\omega$ and find $\rho_\lambda(\lambda)$. Construct a formula giving the 'number of modes in …' by multiplying with suitable quantities.

Example: the number of atoms of mass m in a volume $\mathrm{d}V$ of a pure substance with mass density ϱ is $\mathrm{d}N = \varrho\,\mathrm{d}V/m$.

(2) Eq.(1.1) contains two polarization states per frequency. If the photon modes are constructed from plane waves with k-vector $\mathbf{k}$, then one may choose two polarization vectors $\mathbf{e}_\mu(\mathbf{k})$, $\mu = 1, 2$. Describe in words the geometrical object formed by these three vectors. If $\hat{\mathbf{k}}$ is the unit vector along $\mathbf{k}$, then prove the formula

$$\sum_{\mu=1}^{2} e_{\mu,i}(\mathbf{k}) e_{\mu,j}(\mathbf{k}) = \delta_{ij} - \hat{k}_i \hat{k}_j = \delta_{ij} - \frac{k_i k_j}{k^2} \tag{1.2}$$

Hint. Remember the completeness relation for an orthogonal basis.

(3) Remember that in thermal equilibrium, each field mode of frequency ω carries an average energy given by the Bose-Einstein formula

$$\bar{E}(\omega) = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_B T} - 1} \tag{1.3}$$

Justify why the energy density of light in thermal equilibrium is given by Planck's blackbody spectrum

$$u_{\mathrm{th}}(\omega) = \bar{E}(\omega)\rho_\omega(\omega) \tag{1.4}$$

and check that this is consistent with other expressions that you can find for the Planck spectrum. Find a plot of the spectrum for the cosmic microwave background and admire the accuracy of Planck's formula.