Einführung in die Quantenoptik II

Sommersemester 2014
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Übungsaufgaben Blatt 5

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Problem 5.1 – Quantum jump correlations (10 points)

Consider a two-level system in the 'incoherent limit' where only populations are nonzero in the density matrix with the master equation

$$\frac{d\rho_{ee}}{dt} = -\gamma\rho_{ee} + R\rho_{gg} \tag{5.1}$$

(1) Give an interpretation of the parameters γ and R. (2) Write down the equation of motion for ρ_{gg}. (3) Calculate the inversion $\langle\sigma_z\rangle$ in the stationary state and make a sketch of the time-dependent function $\langle\sigma_z(t)\rangle$ for your favorite initial condition. (4) Derive, in the spirit of the regression formula, a differential equation for the correlation function

$$C_z(t) = \lim_{t'\to\infty} \langle\sigma_z(t'+t)\sigma_z(t')\rangle \tag{5.2}$$

(5) Make a sketch of $C_z(t)$ and of the corresponding spectrum. (6) Give a physical interpretation of $C_z(t)$ using the language of 'quantum jumps' between the states $|g\rangle$ and $|e\rangle$.

The solution to Eq.(5.1) is a sum of a constant and a damped exponential.

Problem 5.2 – Easy Lamb shift (10 points)

A model for the polarizability of a two-level atom that is often used is of the form

$$\alpha(\omega) = \frac{2\omega_A d_{ge}^2/\hbar}{\omega_A^2 - \omega^2} \tag{5.3}$$

(1) Make a sketch of the real and imaginary part of $\alpha(\omega)$ vs. frequency. Check that α/ε_0 has the units of a volume. (2) Recall that the polarizability gives the electric dipole moment in an electric field, $\langle d\rangle = \alpha E$, and justify in words the 'effective interaction energy'

$$U = -\frac{\alpha}{2}E^2 \tag{5.4}$$

Why is this a somewhat loosely defined quantity? (3) Calculate the energy shift in the vacuum state of the electromagnetic field using the spectral expansion

$$\langle \alpha(\omega) E^2 \rangle = \int_0^\infty \mathrm{d}\omega \, \alpha(\omega) \rho_\omega(\omega) \frac{\hbar\omega}{2\varepsilon_0} \,, \qquad \rho_\omega(\omega) = \frac{\omega^2}{\pi^2 c^3} \tag{5.5}$$

where the vacuum mode density $\rho_\omega(\omega)$ was taken from Eq.(1.1). (4) Check that the integral (5.5) is divergent in the UV. (5) One way to extract a finite result is to subtract from the polarizability the one corresponding to a free electron,

$$\alpha_0(\omega) = -\frac{2\omega_A d_{ge}^2/\hbar}{\omega^2} \tag{5.6}$$

and to cut off the integral at a value $\hbar\omega_c \sim mc^2$ where m is the electron mass. Check that the leading order term for the energy shift is of the order

$$\langle U \rangle \sim \frac{(\omega_A/c)^3 d_{ge}^2}{\varepsilon_0} \log \frac{\omega_c}{\omega_A} \tag{5.7}$$

and give a numerical estimate of this expression for the hydrogen atom. Experimentally, this energy shift was determined for the hydrogen atom as a differential shift between the states 2s and 2p, with a value $\mathcal{O}(h \cdot 1000\,\mathrm{MHz})$.

Hans Bethe telling his way of figuring out this estimation at 'Web of Stories': http://webofstories.com/play/4569