Photonen und andere Quasiteilchen

Sommersemester 2015

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Problem 2.1 – Bulk plasmons in metals (10 points)

In the lecture, we have seen how the Coulomb law is formulated in macroscopic electrodynamics. This law can be used to link a macroscopic charge distribution ρ with the electric potential ϕ that it generates. (1) Show that in Fourier space, we have

$$\phi(\mathbf{k}, \omega) = -\frac{\rho(\mathbf{k}, \omega)}{k^2 \varepsilon(\omega)} \tag{2.1}$$

and plot the real and imaginary parts of this 'response function' as a function of frequency ω for the Drude dielectric function (σ is the DC conductivity)

$$\varepsilon(\omega) = \varepsilon_0 + \frac{\mathrm{i}\sigma}{\omega(1 - \mathrm{i}\omega\tau)} \tag{2.2}$$

Result: a peak at the bulk plasmon frequency $\omega_p^2 = \sigma/(\varepsilon_0 \tau)$.

(2) Phenomenological models of relaxation processes sometimes create confusion. For example, is the scattering of electrons in the Drude model consistent with charge conservation? Some time ago, colleagues have argued on the basis of the equation of charge conservation

$$\partial\rho/\partial t = -\nabla \cdot \mathbf{j} \tag{2.3}$$

that in a medium with conductivity σ, the charge density decays to zero on a time scale given by ε_0/σ [Eqs.(13, 14) in *J. Phys. A* **40** (2007) 13485]. (3) Look up numbers in the web for a conductor like gold or silver and check that this would be an extremely fast relaxation process – too fast to be true and consistent in fact.

(4) Try to convince your colleagues that the Drude scattering time τ gives the right order of magnitude for charge relaxation (and is slower) by going back to the way we derived the Drude model in the lecture:

$$\partial\mathbf{j}/\partial t \approx \varepsilon_0 \omega_p^2 \mathbf{E} - \mathbf{j}/\tau \tag{2.4}$$

Problem 2.2 – Surface plasmons (10 points)

In a homogeneous medium with (dimensionless) dielectric function ε, the dispersion relation for transverse waves is

$$k^2 = \varepsilon(\omega)\varpi^2 , \qquad \varpi = \omega/c \tag{2.5}$$

(1) Recall the meaning and orientation of the vectors **k**, **E**, and **H** in a transverse wave.

A surface plasmon is a transverse excitation of the field with a complex k-vector. Consider a magnetic field of the form

$$\mathbf{H}(\mathbf{r}, t) = H_y \hat{\mathbf{y}} \exp(\mathrm{i}qx - \kappa z - \mathrm{i}\omega t) \tag{2.6}$$

where you may add the complex conjugate to make the field real. (2) This field describes a quasi-particle with momentum $\hbar q$ along the surface and energy $\hbar\omega$. Make a sketch of the field distribution. (3) Check that for $z > 0$, this is an acceptable solution to the wave equation in vacuum provided the decay constant κ is given by

$$q^2 - \kappa^2 = \varpi^2 , \qquad q^2 - \kappa_m^2 = \varepsilon(\omega)\varpi^2 \tag{2.7}$$

Below the metal surface ($z < 0$), the surface plasmon is described by a magnetic field

$$\mathbf{H}(\mathbf{r}, t) = H_y \hat{\mathbf{y}} \exp(\mathrm{i}qx + \kappa_m z - \mathrm{i}\omega t) \tag{2.8}$$

(4) Argue that the same amplitude H_y appears in both sides of the interface.

(5) Show that the electric field lies in the xz-plane and is elliptically polarized. Find out the matching conditions for the electric field at $z = 0$. (6) Show that the dispersion relation for the surface plasmon mode can be written as

$$\kappa = -\kappa_m/\varepsilon(\omega) \tag{2.9}$$

and conclude that such a mode can only appear when $\varepsilon(\omega) < 0$ (apply this in practice for the real part). (7) Take the formal limit $c \to \infty$ ('non-retarded' or 'quasi-static' fields) and show that $\kappa \to q$, $\kappa_m \to q$, and the dispersion relation becomes $\varepsilon(\omega) = -1$. Solve this equation for the Drude model.

(8) In the general ('retarded') case, one finds from Eq.(2.9) the dispersion relation

$$q^2 = \varpi^2 \frac{-\varepsilon(\omega)}{-\varepsilon(\omega) - 1} \tag{2.10}$$

Discuss (for real ω) the real and imaginary parts of q and make a plot. Compare to the 'light line' $q = \varpi$ and to the result for a lossless dielectric function

$$\varepsilon(\omega) = \varepsilon_b - \omega_p^2/\omega^2 \tag{2.11}$$

where ε_b is the 'background permittivity' given by the response of bound electrons in the metal.