Photonen und andere Quasiteilchen

Sommersemester 2015
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Problem 7.1 – Quantum fields and particle number (10 points)

Quantum field theory provides the basic machinery for elementary particles. Among the key concepts are field operators. Their non-trivial commutators are responsible for the appearance of discrete numbers of particles. Here is a simple example. Consider the particle number operator

$$\hat{N} = \int \mathrm{d}V\, \psi^\dagger(x)\psi(x) \tag{7.1}$$

where the annihilation $\psi(x)$ and creation operators $\psi^\dagger(x)$ satisfy the equal-time commutation relation:

$$\big[\psi(\mathbf{r},t),\, \psi^\dagger(\mathbf{r}',t)\big] = \delta(\mathbf{r}-\mathbf{r}') \tag{7.2}$$

(i) Show that (at equal times)

$$\big[N,\, \psi^\dagger(x)\big] = \psi^\dagger(x) \tag{7.3}$$

and interpret this equation by applying both sides to the vacuum state.

(ii) Repeat this construction for fermions where instead of Eq.(7.2), we have the anti-commutator

$$\big\{\psi(\mathbf{r},t),\, \psi^\dagger(\mathbf{r}',t)\big\} = \psi(\mathbf{r},t)\,\psi^\dagger(\mathbf{r}',t) + \psi^\dagger(\mathbf{r}',t)\,\psi(\mathbf{r},t) = \delta(\mathbf{r}-\mathbf{r}') \tag{7.4}$$

Problem 7.2 – Sizes of particles (10 points)

(i) Recall the size of a typical nucleus. The common explanation for nuclear forces is based on the exchange of pions π^0 and $\pi^\pm$. Look up the masses m_π of these pions and compare their Compton wavelength $\lambda = \hbar/(mc)$ to the size of the nucleus.

(ii) Find information about the Lamb shift in the hydrogen atom and how it depends on the size of the proton. Compare to experimental data for the Lamb shift in positronium (an electron bound to a positron) and check whether the accuracy is sufficient to measure the size of the electron.

Problem 7.3 – Vacuum polarization (10 points)

Look up the so-called Euler-Heisenberg Lagrangian for the electromagnetic field and find an estimate for which laser fields the 'permittivity of vacuum' significantly differs from ε_0.