Einführung in die Quantenoptik II

Sommersemester 2014
Carsten Henkel

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Problem 1.1 – Laser feeling (10 points)

Please look up the typical elements of a laser: a cavity for the laser mode, an active medium, a mechanism for pumping the medium, an optical system to shape the laser beam.

Look up typical numbers for a laser:

(1) the product ('quality factor' Q) between the resonance frequency ω_c and the 'photon lifetime' τ in the cavity. In the lecture, we shall use $\kappa = 1/\tau$ as 'cavity decay rate'.

(2) the ratio between the input power (needed for pumping the active medium) and the optical output power. This 'efficiency' η is often not very large – this is one of the reasons why laser-induced fusion is probably not a very efficient way of producing energy, for example.

(3) the product of cavity lifetime τ and the flux of photons (photons per second) in the laser beam. We shall see that this is a good estimate for the photon number $\langle n \rangle$ in the cavity. Try to find an estimate. For a 'microlaser' or 'micro maser' (experiments by Nobel prize winner Serge Haroche in Paris), this number is small.

(4) try to find from a 'laser catalogue' information about the fluctuations (stability) of the laser beam power. What physical quantities are used to describe this? We shall be interested in the lecture in the standard deviation Δn of the photon number inside the laser cavity: think how one could translate the catalogue information into the number Δn.

Problem 1.2 – Electric dipole operators of atoms (10 points)

We have introduced in the lecture the transition dipole matrix element

$$\mathbf{d}_{eg} = -e \int \mathrm{d}^3 x \, \psi_e^*(\mathbf{x}) \, \mathbf{x} \, \psi_g(\mathbf{x}) \tag{1.1}$$

where $-e$ is the electron charge, and ψ_e and ψ_g are electronic wave functions. Explain why this formula is only valid for a "one-electron atom" like Hydrogen.

Look up the wave functions (orbitals) for the nlm states of Hydrogen for $n = 1, 2$. Remember the integral

$$\int_0^\infty \mathrm{d}x \, x^n \, \mathrm{e}^{-x} = n! \, , \qquad n = 0, 1 \ldots \tag{1.2}$$

(a) Check that for the states $g = 1s$ and $e = 2s$, $\mathbf{d}_{eg} = 0$.
(b) Check that for $g = 1s$ and $e = 2p_i$ ($i = x, y, z$), the vector $\mathbf{d}_{eg}$ is parallel to the i-axis.
(c) Check that for $g = 1s$ and $e = 2p_m$ ($m = \pm 1$), $\mathbf{d}_{eg}$ is complex, lies in the xy-plane, and changes by a phase factor $\mathrm{e}^{\pm \mathrm{i} m \varphi}$ if it is rotated by an angle φ around the z-axis.
(d) Compute the transition dipole

$$\mathbf{d}_{2p_z 1s} = -e a_0 \hat{\mathbf{z}} f \tag{1.3}$$

where a_0 is the Bohr radius, $\hat{\mathbf{z}}$ is the unit vector along the z-axis, and f is a dimensionless number (whose value is the goal of the problem).