Einführung in die Quantenoptik II

Sommersemester 2014
Carsten Henkel

Übungsaufgaben Blatt 2

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Problem 2.1 – Journal templates in physics (8 points)

Scientific journals propose to their authors "templates" for writing papers. Check out the journals listed below and find their templates on the web.

Each student takes a different template and completes it with the following information: Title, author, affiliation, abstract and bibliography with three entries. If you need inspiration, just produce a fake copy of the paper by Einstein, Podolski and Rosen (1935) about the incompleteness of quantum mechanics. Print out the result and hand it in with the rest of your problem solutions. For one of the problems in this semester, you will be asked to hand in a solution in a similar electronic form (this time under your name).

Nature Photonics, Europhysics Letters, Optics Letters, Physical Review A, Journal of Physics B, European Physical Journal D, Journal of Optics A, Journal of modern Optics, Optics Communications, Journal of the Optical Society of America B, Annalen der Physik (Berlin), Annals of Physics (N.Y.)

[Bonus points if your bibliography is "correctly" formatted.]

Problem 2.2 – Two-level language (6 points)

In the lecture, we have seen a few observables for a two-level atom. In this problem, you construct the matrix representation of these operators. Remember that one always has to specify how the basis vectors are chosen. My personal convention is the mapping

$$|\mathrm{e}\rangle \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\mathrm{g}\rangle \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.1}$$

('The excited state is above the ground state.') Give matrix representations for the two-level operators introduced in the lecture

$$\pi_e, \quad \pi_g, \quad \sigma = |\mathrm{g}\rangle\langle\mathrm{e}|, \quad \sigma^\dagger \tag{2.2}$$

Take an arbitrary hermitian 2×2 matrix A and justify why it can be written in the form

$$A = a_e \pi_e + a_g \pi_g + a_{ge} \sigma + a_{eg}^* \sigma^* \tag{2.3}$$

where the numbers a_e are ... (real? complex? quaternions?). Check whether the list (2.2) of two-level operators (call them $S_1 \dots S_4$ for simplicity) is orthogonal in the sense of the scalar product

$$(S_i, S_j) := \operatorname{tr}\left(S_i^\dagger S_j\right) \tag{2.4}$$

(Check first that this is indeed a scalar product.)

Problem 2.3 – Rate equations for a laser (6 points)

A simplified model for a laser is based on rate equations. The basic quantities are the atom numbers $n_{e,g}$ in the excited state and in the ground state, and the (average) number N of photons.

The photon number evolves according to

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\kappa N + \gamma n_e + \gamma N(n_e - n_g) \tag{2.5}$$

(1) Explain the meaning of the three terms. Recall that κ is the cavity loss rate and $1/\gamma$ the lifetime of the excited state.

(2) Explain the rate equations for the atom numbers

$$\begin{aligned} \frac{\mathrm{d}n_e}{\mathrm{d}t} &= -\gamma(N+1)n_e + \gamma N n_g \\ \frac{\mathrm{d}n_g}{\mathrm{d}t} &= -\gamma N n_g + \gamma(N+1)n_e \end{aligned} \tag{2.6}$$

(3) Show that the stationary state of these rate equations is $N = 0 = n_e$, while n_g can be taken as a constant (the total number of atoms, say N_A).

(4) Add a 'pumping rate' $+\lambda_e$ (not a wavelength!) to $\mathrm{d}n_e/\mathrm{d}t$ and solve for the stationary state.

This is still not a 'lasing state' because the medium is not inverted, $n_e < n_g$.