Theoretical Physics
- Introduction to General Relativity (Summer 2016) -

Problem set No 2

Emission 06.04.16 – Digestion 08.04.16

▷ Problem 1 (Clock synchronization)

Synchronization of clocks in an inertial system is based on an empirical fact and a definition:

Fact: On any closed loop, the ratio of path length and travel time of light is a loop-independent constant

$$\frac{\text{Length of loop } L}{\text{duration of round-trip } T} := c = const. \tag{1}$$

Convention: (Definition of "Simultaneity") Two flash-events are called to happen at the same time if their light-fronts meet in the middle.

Flash event? ... is realized, if you push the flash-button of your camera (because there is not enough ambient light). Ideal point flashes emit a spherical light wave, the front of which expands with the "speed of light" (see the note below, however). Never mind that there are no ideal point flashes on the market ...

Note: Do *not* read the fact "the speed of ligth is a constant". It is – but this comes only after some logical inferences which also involve the convention.

(a) Show that simultaneity obeys the axioms of an equivlance relation on the set of all events. (if P is simultaneous with Q, and Q is simultaneous with R, then P is simultaneous with R).

(b) Describe a procedure by which a given pair of clocks in an inertial system can be synchronized.

Note: *Now* you may state, that the speed of light is a constant!

Remark: Part (a) sounds kind of trivial, but it is not. You must show, that under the assumption that the flash-fronts of P and Q meet in the middle, and the flash-fronts of Q and R meet in the middle (although at a different time), then also the flash-fronts of P and R meet in the middle (at still another time). The difficulty is, that you can not use the every-day notion of velocity, but only the "fact" as it is formulated (for "fact" you only need a ruler and *one* clock. The every-day notion of velocity is inappropriate, because it is either based on a traveling clock – which for light is impossible – or two clocks, one at start, one at finish, which must be synchronized. Yet synchronization is only possible once simultaneity has been proven to constitute an equivalence relation, that is (a).

▷ Problem 2 (Real clock precision)

The time standard on time-like worldlines is kept by clocks which have no extension – they are just points. Unfortunately, real clocks have spatial extension. In a gravitational field, the physical mechanism of such clocks is subject to tidal forces. Estimate the influence the tidal forces have on the functioning of an atomic clock.

▷ Problem 3 (Merry-go-round and the Fountain of Youth I)

A merry-go-round rotates with angular frequency Ω. Show that a passenger, who is placed at distance R to the axis of the merry-go-round, reads from his wristwatch a period of rotation

$$T_{\mathrm{rot}} = \sqrt{1 - \frac{\Omega^2 R^2}{c^2}} T_{\mathrm{lab}} \tag{2}$$

where $T_{\mathrm{lab}} = 2\pi/\Omega$ is the period of rotation in the laboratory frame.

In view of () – in what respect would a ride on a merry-go-round be an appropriate measure for anti-aging therapy in a "fountain of youth"?

<u>Remark:</u> See next problem set for the suitability of centrifugal forces for the fountain of youth ...

▷ Problem 4 (Hyperbolic motion)

Dr. Rindler moves with constant acceleration (as measured in his rest frame). He carries a good clock along (proper time τ) which allows him to determine the duration of any physical process.[1] As described with respect to an inertial system with coordinates t, x, y, z you may assume that his motion is in the x-direction.

(a) Show that in inertial coordinates Dr. Rindler's worldline may be parametrized

$$t(\tau) = \frac{c}{g} \sinh(g\tau/c) + t_0 \,, \tag{3}$$

$$x(\tau) = \frac{c^2}{g} \cosh(g\tau/c) + x_0 - \frac{c^2}{g} \,, \tag{4}$$

where τ is Rindler's proper time, $\tau = 0$ corresponding to coordinate time value t_0 , and x_0 Rindler's position in the inertial system at $t = t_0$.

In special relativity, a motion with constant acceleration is sometimes called *hyperbolic motion.* Any idea why?

(b) Convince yourself that Rindler's velocity, as measured from the inertial system, is given by

$$v(t) = \frac{(t - t_0)g}{\sqrt{1 + [(t - t_0)g/c]^2}} \tag{5}$$

[1] Assume the earth to be flat, and thus its gravitational field to be homogeneous. Standing on earth, your impression is that the gravitational force pulls you downwards (the $-x$-direction). According to the equivalence principle, your impression is that of a commander of a space ship, which accelerates relative to an inertial system upwards.

and thus after sufficinetly long time

$$|t - t_0| \gg c/g : \qquad |v(t)| \approx c\,. \tag{6}$$

For the earth gravitational acceleration $g = 9,81\text{m}/\text{sec}^2$ – what does it mean "sufficiently long" with respect to (i) the inertial system, (ii) the rest frame of Rindler?

(c) Show that in the (ct, x)-coordinates of the inertial system, the Rindler manifold of equal time events are straight lines which intersect at a spacetime event with coordinates $t_0, x_0 - c^2/g$. Without loss of generality, you may choose $t_0 = 0$ and $x_0 = c^2/g$, such that O is assigned inertial coordinates $(0, 0)$. In the xct-plane, plot the Rindler's worldline and a few of his equal time slices incl. slices where $\tau = \pm\infty$.

(d) The event O in (c) marks a singularity in the Rindler's coordinates: for him, O is of eternal duration. Convince yourself, that from the Rindler's point of view, that singularity is always at spatial distance c^2/g. What distance would that be for the earth gravitational acceleration $g = 9,81\text{m}/\text{sec}^2$?

(e) Rindler's $\tau = \pm\infty$ slices partition spacetime into four different regions. Discuss from which events Rindler can receive information, and which events can receive information from Rindler. Are there events, which are absolutely inaccessible by Rindler? If yes – what about time ordering of these events in Rindler coordinates?

(f) Rindler introduces coordinates $(c\tau, \bar{x})$ using regularly spaced spatial marks to his left and to his right, which are relative to Rindler at rest, choosing $\bar{x} = 0$ for his own position. As time standard he uses his proper time τ. Give the coordinate transformation from intertial coordinates to the Rindler coordinates.

(g) Another story: Two rockets, stacked vertically in free space, and connected by a thin rope, start with equal acceleration g. The rockets engine are such, that during boost operation, the acceleration as felt by the rocket system is constant. After a certain time T (as measured from an inertial system), there is Brennschluss for both rockets. Question: After Brennschluss – is the connecting rope still intact or not? (you may assume that the rope is infinitely thin and has no inertia). Another question: what is the difference with (f)?