Einführung in die Quantenoptik II

Sommersemester 2016
Carsten Henkel

Übungsaufgaben Blatt 5 Ausgabe: 10. Juni 2016
Abgabe: 22. Juni 2016

Problem 5.1 – Fokker-Planck model of a laser (10 points)

In the lecture, we have found the following Fokker-Planck equation for the P-function $P(\alpha, t)$ (in polar coordinates, $\alpha = r\,\mathrm{e}^{\mathrm{i}\varphi}$)

$$\frac{\partial P}{\partial t} = \omega_L \frac{\partial P}{\partial \varphi} + \frac{1}{2r}\frac{\partial}{\partial r} r^2(\kappa - G(r^2))P + \frac{1}{4r}\left(\frac{\partial}{\partial r} r G(r^2)\frac{\partial}{\partial r} + \frac{G(r^2)}{r}\frac{\partial^2}{\partial \varphi^2}\right) P. \quad (5.1)$$

where κ, $G(r^2)$ are the loss and gain rates.

(i) Check that the stationary state for the following gain saturation model

$$G(r^2) = \frac{G_0}{1 + Br^2} \quad (5.2)$$

is given by the shifted Gaussian

$$P_{\mathrm{ss}}(r) = N \exp\left[-\frac{B\kappa}{2G_0}(r^2 - \bar{n})^2\right], \qquad \bar{n} = \frac{G_0 - \kappa}{\kappa B} \quad (5.3)$$

where N is a normalization factor.

(ii) Look up the Chapter 18.6 in Mandel & Wolf's book and pursue the analogy to the Schrödinger equation. Make the following *Ansatz*

$$P(r, \varphi, t) = [P_{\mathrm{ss}}(r)]^{1/2} P_m(r)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\lambda t} \quad (5.4)$$

and find a Schrödinger-like equation for $P_m(r)$. Trace the 'effective potential' for $m = 0, \pm 1$ and comment on the sign of the eigenvalues λ. How do the eigenvalues change from $m = 0$ to $m = \pm 1$?

Problem 5.2 – Lorenz model (10 points)

In Chapter 18.8 of Mandel & Wolf, the Lorenz model is introduced as a simplified version of the dynamics of a laser coupled to its active medium. (i) Check

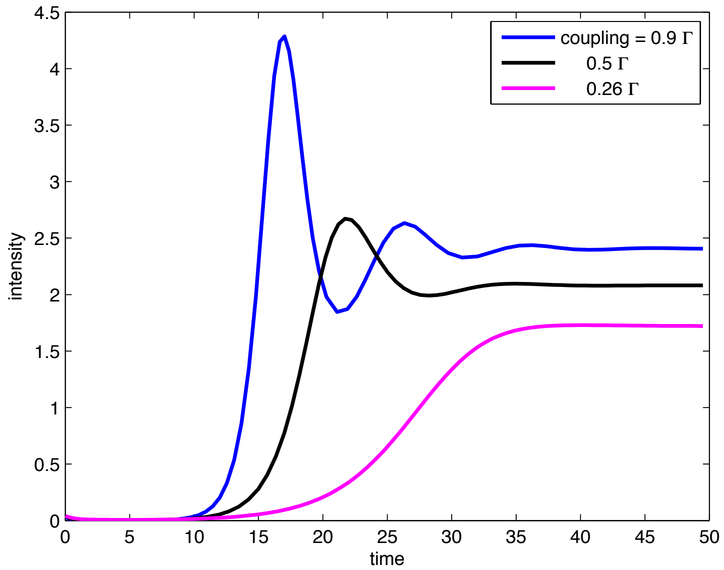

Figure 5.1: Transient laser intensity after onset, based on a three-component Bloch-Lorenz model. The coupling is proportional to the parameter G_0 in Eqs.(5.7).

from the Bloch equations discussed in the lecture, that for the (complex) polarization P of the medium and its inversion (real) N, the following equations are reasonable

$$\partial_t P = (\mathrm{i}\Delta - \Gamma)P - \mathrm{i}gN\mathcal{E} \tag{5.5}$$

$$\partial_t N = -\gamma(N - N_s) + g\,\mathrm{Im}[P^*\mathcal{E}] \tag{5.6}$$

$$\partial_t \mathcal{E} = -\kappa\mathcal{E} + \mathrm{i}G_0 P \tag{5.7}$$

where in the last line, we also added the dynamics of the (complex) laser amplitude $\mathcal{E}$ (slowly varying, omitting the carrier $\mathrm{e}^{-\mathrm{i}\omega_L t}$).

(ii) Show that by using suitable units for P, N, $\mathcal{E}$, and time, the number of parameters can be reduced to three. Show that with the phase transformation $P(t) = P'(t)\,\mathrm{e}^{-\mathrm{i}\delta t}$ and similarly for $\mathcal{E}$, the detuning Δ can be 'distributed' in a symmetric way.

(iii) Write a program that solves these equations numerically and try to find parameters (and initial data) that qualitatively reproduce Fig.5.1.

Hint. For the 'spikes', you need a large inversion with a slow relaxation. A rapid damping of the polarization does not seem problematic.