Aufgabe 3.1 – Bulk and surface plasmon resonances (10 Punkte)

(i) Look up the plasma frequencies ω_p of metals like gold, silver, copper. Typical values are in the blue to ultraviolet. Check whether two definitions are consistent: the formula from the (conduction) electron density n

$$\omega_p^2 = \frac{e^2 n}{\varepsilon_0 m} \tag{3.1}$$

(m is the conduction electron mass) and the zero of the dielectric function, $\varepsilon(\omega_p) \approx 0$. One of the first Google hits is the web site 'wave-scattering.com' with a table with plasma frequencies.

(ii) The energy loss of electrons that are shot through a medium is proportional to the imaginary part of the 'screening factor' $-1/\varepsilon(\omega)$. Using a Drude model for the dielectric function
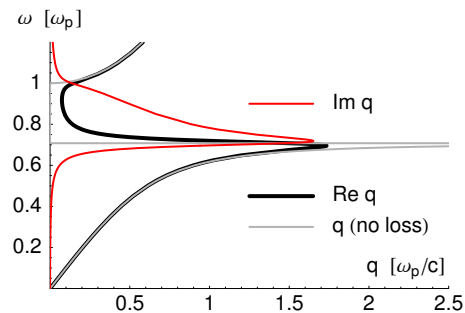
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\gamma)}, \tag{3.2}$$

make a plot of the electron loss spectrum. What is the interpretation of the peak at the plasma frequency $\omega \approx \omega_p$?
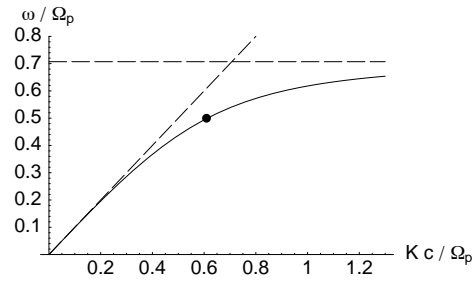
(iii) Plot the real and imaginary parts of the surface plasmon wave vector as a function of frequency

$$q = \frac{\omega}{c}\sqrt{\frac{-\varepsilon(\omega)}{-\varepsilon(\omega) - 1}} \tag{3.3}$$

(a kind of complex dispersion relation). Your result may look like the picture on the right. Give an interpretation of the essential features.

Aufgabe 3.2 – Exciting a surface plasmon (10 Punkte)

(i) The dispersion relation of a surface plasmon at a metal-vacuum interface is sketched on the left (K is the same as q in Problem 3.1, $\Omega_p = \omega_p$.) Add to this picture the dispersion relation of light in vacuum (this is typically called the 'light cone') and in a medium with index $n > 1$ ('glass') Fix a frequency $\omega \sim 0.5\Omega_p$ and find the angle of a light beam in the glass that can excite the surface plasmon – apart from the problem how to bring together these three media.

(ii) Look up the keywords 'Otto' and 'Kretschmann configuration' for the excitation of a surface plasmon on a metallic film. Split with your fellow students in two groups and choose one of the configurations: on which side of the metallic layer does the excited plasmon 'live'?

(iii) Surface plasmons can also be generated with a laser beam and a grating written onto the surface. Make a sketch how the diffracted wave vectors match with the plasmon's dispersion relation.

(iv) [5 Bonus points] Consider the case of a 'rough surface' that may be considered as many gratings with a wide distribution of grating vectors. If you generate a plasmon on a not-so-smooth metallic surface, one can 'see' it because it scatters light. What kind of angular pattern for the stray light do you expect?