Optik an Grenzflächen und Nanostrukturen Sommersemester 2017

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Aufgabe 4.1 – Surface plasmon dispersion (10 Punkte)

In Raether's book on surface plasmons, you can find this Figure, showing schematically the dispersion relation $\omega(k)$. The up-bending at large k is the topic of this exercise.

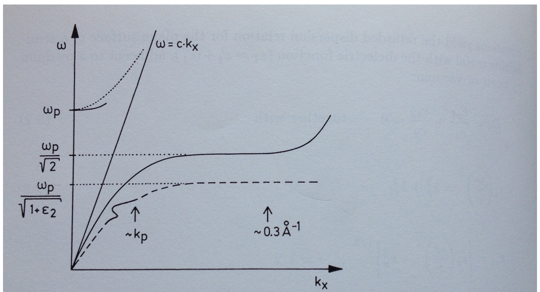


Fig. 2.2. The dispersion relation of nonradiative SPs (——), right of the light line $\omega = ck_x$; the retardation region extends from $k_x = 0$ up to about $k_p = 2\pi/\lambda_p$ (λ_p plasma wavelength). The dashed line, right of $\omega = ck_x$, represents SPs on a metal surface coated with a dielectric film (ε_2). Left of the light line, $\omega(k_x)$ of the radiative SPs starts at ω_p (——). (The *dotted line* represents the dispersion of light in a metal: $\omega/k_x = c/|\varepsilon_1'|^{1/2}$ or in the case of free electrons $\omega^2 = \omega_p^2 + c^2 k_x^2$.) The slight modulation in the dashed dispersion curve comes from an eigen frequency in a monomolecular dye film deposited on a Langmuir-Blodgett film (ε_2). The latter depresses $\omega_p/\sqrt{2}$ to $\omega_p/\sqrt{1+\varepsilon_2}$

(i) Compare the surface plasmon wavelength (parallel to the surface) where the up-bending appears, to the plasma wavelength $\lambda_p = 2\pi/k_p = 2\pi c/\omega_p$ and speculate why this effect is often attributed to 'microscopic physics'.

(ii) Since the plasmon dispersion appears for large k (small scales), let us try a simplified description where retardation is neglected. The light field is described by the electric potential $\phi(\mathbf{r})$ alone (no vector potential). If this potential varies like $\mathrm{e}^{\mathrm{i}k_x x}\phi(z)$ outside the metal (i.e., in the region $z \geq 0$), show that $\phi(z) \sim \mathrm{e}^{-|k_x|z}$. Similarly, you may assume that $\phi(z) \sim \mathrm{e}^{|k_x|z}$ inside the metal ($z \leq 0$).

(iii) Start with the Ansatz for the potential

$$\phi(x,z) = \begin{cases} \phi_0\, \mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{-|k_x|z} & z \geq 0 \\ \phi_m\, \mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{|k_x|z} & z \leq 0 \end{cases} \tag{4.1}$$

and show that the continuity of ϕ and of εE_z gives the following dispersion equation for the surface plasmon:

$$k\varepsilon_0 = -k\varepsilon_m\,, \qquad k = |k_x| \tag{4.2}$$

or $\omega = \omega_p/\sqrt{2}$ for a Drude metallic dielectric function (no losses, no background polarisation).

(iv) On short scales, the electric charge density at a metallic surface may also oscillate in such a way that a *surface polarisation* P_s is built up. This corresponds to an area density of dipole moments (oriented perpendicular to the surface, in other fields also known as 'double layer'). By analogy to a parallel-plate capacitor (*Plattenkondensator*), such a surface polarisation translates into a '*jump condition*' for the electric potential:

$$\lim_{z \searrow 0} \phi(z) = \lim_{z \nearrow 0} \phi(z) + \frac{P_s}{\varepsilon_0} \tag{4.3}$$

where the limits are understood as 'coming from outside the metal' and 'coming from inside the metal' (while points within the surface polarization are 'forbidden'). Make a sketch of the electric potential. Show that we get the system of equations for the surface plasmon:

$$\phi_0 = \phi_m + \frac{P_s}{\varepsilon_0}\,, \qquad k\varepsilon_0\phi_0 = -k\varepsilon_m\phi_m \tag{4.4}$$

(v) To solve these equations, we need a relation between the surface polarisation and the electric field. The simplest local *Ansatz* is a linear response to the 'inner field'

$$P_s = \varepsilon_0 \alpha \lim_{z \nearrow 0} E_z \tag{4.5}$$

with a complex coefficient α, the so-called 'surface polarisability'. Check that α has the dimensions of a length: since Feibelman's work [Phys. Rev. B **40** (1989) 2752], it is interpreted as the 'centroid' (*Schwerpunkt*) of the electronic charge density of the metallic surface. Show that in terms of α, one gets with a lossless Drude model for ε_m the dispersion relation

$$\omega_{\rm sp}(k) \approx \frac{\omega_p}{\sqrt{2 - k\alpha}} \tag{4.6}$$

One typically considers that this simple model is only meaningful when α is small: expand the dispersion relation and estimate the group velocity $\partial\omega_{\rm sp}/\partial k$ for the surface plasmon.

Aufgabe 4.2 – Graphene plasmons (10 Punkte)

Graphene is a two-dimensional material made from a one-atom thick layer of carbon atoms, arranged in a honeycomb lattice. Because of delocalised electronic states, it has a very high in-plane conductivity $\sigma = J/E_\parallel$ where J is the surface current density and $E_\parallel$ the in-plane electric field. We construct in this problem a surface plasmon whose field is peaking right in the graphene plane (located at $z = 0$).

(i) For a free standing graphene film, consider a magnetic field (all fields oscillate $\sim {\rm e}^{-{\rm i}\omega t}$, $H(+0) = \lim_{z\searrow 0} H(z)$)

$$H_y(x,z) = \begin{cases} H(+0)\,{\rm e}^{{\rm i}k_x x - \kappa z}\,, & z > 0 \\ H(-0)\,{\rm e}^{{\rm i}k_x x + \kappa z}\,, & z < 0 \end{cases} \tag{4.7}$$

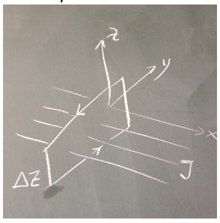
and show that its jump at $z = 0$ is proportional to the surface current density

$$H(+0)\,{\rm e}^{{\rm i}k_x x} - H(-0)\,{\rm e}^{{\rm i}k_x x} = -J_x \tag{4.8}$$

Solution. In the Ampère–Maxwell equation

$$\nabla \times \mathbf{H} = \mathbf{j} + \partial_t \mathbf{D}\,,$$

we need the ('external') current density to fix the jump in the magnetic field. (This is similar to the jump in the normal electric field in $\nabla \cdot \mathbf{D} = \rho$ when a surface charge is present.) A geometric argument



is based on Stokes' theorem and a small area $\Delta a = \Delta y \Delta z$ that is 'pierced' by the graphene sheet. The area normal is along the x-direction, parallel to the surface current $\mathbf{J}$. The integral over $\nabla \times \mathbf{H}$ is transformed into a closed line integral

$$\int (\nabla \times \mathbf{H}) \cdot \mathrm{d}\mathbf{a} = \oint \mathbf{H} \cdot \mathrm{d}\mathbf{s} \approx [-H_y(+0) + H_y(-0)]\,\Delta y$$

where in the last expression, the limit $\Delta z \to 0$ was taken. The integral over the current density $\mathbf{j}$ gives in the same limit the sheet current

$$\int \mathbf{j} \cdot \mathrm{d}\mathbf{a} = J_x \Delta y$$

(because this current is localised, like a δ-function in the interior of the area). Finally, for the displacement current $\partial_t D$,

$$\int \partial_t \mathbf{D} \cdot \mathrm{d}\mathbf{a} = \overline{\partial_t \mathbf{D}}\,\Delta y \Delta z$$

we argue that $\partial_t \mathbf{D}$ is bounded so that the average value theorem can be applied. (After all, $\mathbf{D}$ is continuous across the graphene sheet.) As a result, this last integral vanishes in the limit $\Delta z \to 0$, and we get

$$-H_y(x, y, +0) + H_y(x, y, -0) = J_x$$

which is Eq.(4.8).

(ii) Evaluating the Ampère-Maxwell equation outside the film, compute the tangential electric field

$$E_x(x,z) = \begin{cases} \dfrac{\mathrm{i}\kappa H(+0)}{\omega \varepsilon_0}\, \mathrm{e}^{\mathrm{i}k_x x - \kappa z}\,, & z > 0 \\ -\dfrac{\mathrm{i}\kappa H(-0)}{\omega \varepsilon_0}\, \mathrm{e}^{\mathrm{i}k_x x + \kappa z}\,, & z < 0 \end{cases} \tag{4.9}$$

Since this field is continuous across the film, we can conclude that $H(-0) = -H(+0)$. Make a sketch of the magnetic and electric fields.

(iii) By using Ohm's law for the surface current J_x, find the dispersion relation

$$\omega = -\frac{\mathrm{i}\sigma}{2\varepsilon_0}\kappa \tag{4.10}$$

For a surface plasmon with weak damping, we thus need an imaginary conductivity. The simplest model is to describe the graphene electrons as a two-dimensional ideal gas with area density n_s and without damping:

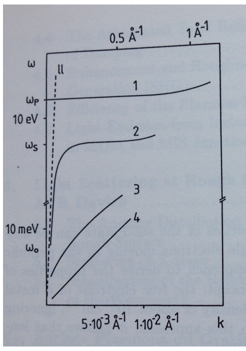


Fig. 1.1. Scheme of the dispersion of plasmons in a solid. *(1)* volume plasmons, *(2)* surface plasmons, *(3)* plasmons in a two-dimensional electron gas, *(4)* plasmons in a one-dimensional system. The upper scale is valid for *(1)* and *(2)*, the scale below for *(3)* and *(4)*. *ll* is the light line

$$\sigma = \mathrm{i}\frac{n_s e^2}{m^* \omega} \tag{4.11}$$

Conclude that the dispersion follows the power law $\omega \sim \kappa^{1/2} \approx k^{1/2}$. This is sketched in Raether's figure shown left (curve no. 3). From the numbers given there, find an estimation of the charge density n_s. (At the time of Raether's book, graphene was not known, of course.)