Einführung in die Quantenoptik II

Sommersemester 2017

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Problem 3.1 – State transformations (12 points)

The squeezer. In the lecture, we have seen that a transformation $S = S(\xi)$ exists (the squeezing operator) such that

$$S^\dagger a S = \mu a + \nu a^\dagger \tag{3.1}$$

with complex parameters μ, ν. (i) Show that $S^\dagger a^\dagger S = \nu^* a + \mu^* a^\dagger$. (ii) If S is unitary, then $[S^\dagger a S, S^\dagger a^\dagger S] = \mathbb{1}$. (iii) This is true provided the squeezing parameters are related by the 'hyperbolic identity': $|\mu|^2 - |\nu|^2 = 1$. An often used parametrisation is $\mu = \cosh \xi$, $\nu = \mathrm{e}^{\mathrm{i}\varphi} \sinh \xi$.

Squeeze and displace. In the lecture, we have sketched states that are both 'coherent' and 'squeezed' (in phase space: ellipses displacement from the origin). What role is played by the ordering of the two operations squeezing S and displacement D

$$SD|0\rangle \neq DS|0\rangle\,? \tag{3.2}$$

(i) Argue that an answer can be found by analysing the operators $S^\dagger DS$ and $D^\dagger SD$. (ii) Using Eqs.(1.5, 3.1), show that $S^\dagger DS$ is another displacement operator D' and check that its parameter $\alpha' = \mu^* \alpha - \nu \alpha^*$. (iv) Choose some values for μ, ν, α and make a sketch of the states $S|0\rangle$, $DS|0\rangle$, $D'|0\rangle$, and $SD'|0\rangle$.

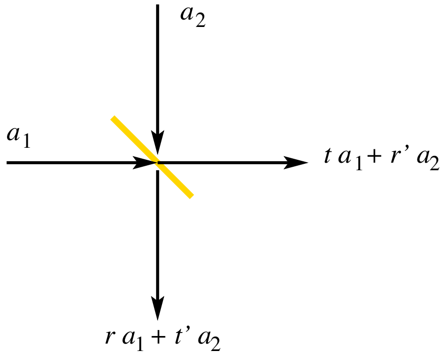
The squeezing transformation can be written in matrix form

$$S^\dagger \begin{pmatrix} a \\ a^\dagger \end{pmatrix} S = \mathsf{M} \begin{pmatrix} a \\ a^\dagger \end{pmatrix} \tag{3.3}$$

and one finds that the relation between the displacement parameters corresponds to the inverse matrix

$$\begin{pmatrix} \alpha' \\ \alpha'^* \end{pmatrix} = \mathsf{M}^{-1} \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix} \tag{3.4}$$

Problem 3.2 – Homodyne measurements (8 points)



In the lecture, we have discussed the beam-splitter sketched in the left picture.

(i) Write down the photon number operator $\hat{n}$ of the beam output to the right, taking for the input beam a_2 a 'bright coherent state'.

(ii) Identify a quadrature operator $X_1(\theta)$ in the operator $\hat{n}$. You may have to find the 'best choice' of r' and t. Keep only contributions proportional to $a_2^\dagger a_2$, a_2, and $a_2^\dagger$.

(iii) Compute the fluctuations $\Delta\hat{n}^2$ of the photon number: they are proportional to the variance $\Delta X_1(\theta)^2$. In this measurement, one can thus check whether the input beam a_1 is squeezed.