Einführung in die Quantenoptik II

Sommersemester 2017
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Übungsaufgaben Blatt 6

Ausgabe: 30. Juni 2017
Abgabe: 12. Juli 2017

Problem 6.1 – Quantum regression (10 points)

In this exercise, you are going to write your first paper. Well, you pretend to do so and take the 'templates' that are provided by scientific publishers for their potential authors.

(1) Look up on the Web the paper templates for physics journals like *European Physical Journal D* (Springer and EDP Sciences), *Journal of Physics B* (Institute of Physics UK), *Physical Review A* (American Physical Society), and *Physics Letters A* (Elsevier). Make a table with links to the template files.

(2) Choose one of the templates and write a paper with title, author name (you), affiliation (Institute of Physics and Astronomy, University of Potsdam), abstract etc. on the following topic: "Quantum Regression". You can take the material from the Lecture and cover the following aspects:

- composite systems and reduced density operators
- master equation for an open system and representation as Hamiltonian evolution of an enlarged system
- correlation functions written in terms of the Heisenberg operators evolving for the larger system
- identify the correlation function as a solution to a master equation with a particular initial value
- apply this technique to the autocorrelation $C_\sigma(\tau) = \langle \sigma^\dagger(\tau)\sigma(0)\rangle$ of a two-level atom that undergoes purely spontaneous decay, but is somehow prepared in a state where $p_e = \langle \sigma^\dagger(0)\sigma(0)\rangle \neq 0$
- give the spectrum corresponding to $C_\sigma(\tau)$ [5 bonus points for a plot in the paper.]

In your literature list, you should cite at least the historical review "The Lax-Onsager regression 'theorem' revisited" by Melvin Lax [*Opt. Commun.* **179** (2000) 463]. Other useful references can also be found there.

Problem 6.2 – Anti-bunching (10 points)

In the lecture, we have found the quantum regression formula for correlation functions

$$\lim_{t\to\infty} \langle A(t')B(t)\rangle = \mathrm{tr}\Big\{ A P(t-t'|B\rho_{\rm ss})\Big\} \tag{6.1}$$

where $P(\tau|\varrho)$ solves the master equation with initial condition $P(0|\varrho) = \varrho$ and $\rho_{\rm ss}$ is the stationary state (density operator) of the system.

(i) This formalism generalizes a master equation to 'skew' objects like $B\rho_{\rm ss}$ that are not, strictly speaking, density operators. Show that also skew objects can be evolved by checking the identity

$$\begin{aligned} B\rho_{\rm ss} &= \frac{1}{4b^*}\Big\{(b+B)\rho_{\rm ss}(b^*+B^\dagger) - (b-B)\rho_{\rm ss}(b^*-B^\dagger) \\ &\quad + \mathrm{i}(\mathrm{i}b+B)\rho_{\rm ss}(-\mathrm{i}b^*+B^\dagger) - \mathrm{i}(\mathrm{i}b-B)\rho_{\rm ss}(-\mathrm{i}b^*-B^\dagger)\Big\} \end{aligned} \tag{6.2}$$

Is each of the terms a density operator? Does the solution of the master equation depend linearly on the initial conditions?

(ii) Consider the excited state projector $\pi_e = \sigma^\dagger\sigma$. Construct from the regression formula a way to compute the time and normal ordered correlation function ($t' \geq t \to \infty$)

$$C(t'-t) = \langle :\pi_e(t')\pi_e(t):\rangle = \langle \sigma^\dagger(t)\sigma^\dagger(t')\sigma(t')\sigma(t)\rangle \tag{6.3}$$

and show that this does *not* involve the skew operator construction of (i). Argue that $C(0) = 0$: this is called 'anti-bunching'. Show that for $t'-t\to\infty$, we have $C(t'-t) \to \langle \pi_e\rangle_{\rm ss}^2$ where $\langle \ldots \rangle_{\rm ss}$ is the average in the steady state $\rho_{\rm ss}$.