Makroskopische Quantenzustände und kalte Gase

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Problem 3.1 – Radiation pressure (10 points)

Light exerts forces on atoms by exchanging both energy and momentum. One prominent example is the radiation pressure force that we shall see in the lecture. In order of magnitude, it is given by

$$F_{\rm rp} = \hbar k \gamma_{e\to g} p_e \tag{3.1}$$

where $k = \omega_{eg}/c$ is the wavenumber of a transition $e \leftrightarrow g$, $\gamma_{e\to g}$ the spontaneous decay rate between the levels, and p_e the probability of finding the atom in the excited state.

(1) Estimate this force for the D-line in sodium and compare it to gravity.

The force arises from the momentum, $\hbar\mathbf{k}$, taken from a laser beam as a photon is absorbed, minus the recoil $\hbar\mathbf{k}_e$ as the atom emits spontaneously a photon. One 'typically' assumes that both momenta have the same length k (why?). The 'spontaneous photons' are emitted with an angular distribution $P(\theta, \varphi)$ given by the emission pattern of the dipole transition driven by the laser. For a linear polarisation, the pattern is (up to a normalisation)

$$\text{lin}: \qquad P(\theta,\varphi) \sim \sin^2\theta \tag{3.2}$$

where θ is the angle relative to the laser polarisation. For circular polarisation:

$$\text{circ}: \qquad P(\theta,\varphi) \sim 1 + \cos^2\theta \tag{3.3}$$

Here, θ is the angle relative to the direction perpendicular to the plane of circular polarisation. (2) Make a sketch of these two angular distributions and compute their normalisations.

(3) Show that the average recoil momentum from to spontaneous emission is zero:

$$\langle \hbar\mathbf{k}_e\rangle = \hbar \int \mathrm{d}\Omega\, P(\theta,\varphi)\mathbf{k}_e = 0 \tag{3.4}$$

Try to find symmetry argument and avoid the evaluation of the integral.

(4) But the variance of the recoil is nonzero, and this is responsible for 'recoil heating'. Find a symmetry-related coordinate system where the 'variance tensor' $\langle k_i k_j\rangle$ is diagonal. Split in groups, choose these diagonal directions and compute

$$\langle k_i^2\rangle = \int \mathrm{d}\Omega\, P(\theta,\varphi) k_{e,i}^2 \tag{3.5}$$

A typical result is $\langle k_i^2 \rangle = \frac{2}{5} k^2$ along a direction perpendicular to the (linear) laser polarisation.

(5) In recent years, people have a found a way to generate a 'directional' emission pattern so that the average recoil momentum $\langle \hbar \mathbf{k}_e \rangle \neq \mathbf{0}$. This can give rise to a high radiation pressure. Look up the key words 'photonic spin-Hall effect', 'spin-orbit interaction of light', or 'chiral waveguide' and try to sketch the idea.

Problem 3.2 – Optical potentials (10 points)

In the lecture, we are going to discuss optical tweezers or 'dipole traps'. They can be modelled by an effective potential of the form

$$V_{\rm dip}(\mathbf{r}) = -\,\mathrm{Re}\,\alpha(\omega) |\mathcal{E}(\mathbf{r})|^2 \tag{3.6}$$

where $\mathcal{E}(\mathbf{r})$ is the complex amplitude of a laser field at frequency ω, using the 'Paris convention':

$$\mathbf{E}(\mathbf{r}, t) = \mathcal{E}(\mathbf{r})\, \mathrm{e}^{-\mathrm{i}\omega t} + \mathrm{h.c.} \tag{3.7}$$

For a focused laser beam, this potential has roughly an anisotropic Gaussian shape.
(1) An often used device is a CO_2 laser that provides huge intensities in the infrared. Look up typical parameters and estimate the sign and the depth of the potential for the sodium D-line. You may use the polarisability of Problem 1.3, Eq.(1.5), or its far-off resonance generalisation

$$\alpha(\omega) = 2\pi\varepsilon_0 \lambda\!\!\!^{-}{}_D^3 \frac{\omega_D \gamma}{\omega_D^2 - \omega^2 - \mathrm{i}\omega\gamma} \approx \frac{\pi\varepsilon_0 \lambda\!\!\!^{-}{}_D^3 \gamma}{\omega_D - \omega - \mathrm{i}\gamma/2} \tag{3.8}$$

(3) In which parameter regime does the dipole potential scale like $1/|\omega_L - \omega_D|$ (which is an often-used rule of thumb)?

(4) Look up the key word 'optical tweezer' and try to compare the magnitude of the optical potentials used there. Note that a tweezer can also trap macroscopic particles like small glass beads.

(5) When two laser beams with opposite direction interfere, they form a standing wave pattern:

$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_1\, \mathrm{e}^{ikz} + \mathcal{E}_2\, \mathrm{e}^{-ikz} \tag{3.9}$$

If the two beams have the polarisation, one gets a simple intensity pattern. Two even more popular configurations are 'lin⊥lin' and 'σ^+/σ^-'. In the second case, $\mathcal{E}_1 \sim \mathbf{e}_x + \mathrm{i}\mathbf{e}_y$ and $\mathcal{E}_2 \sim \mathbf{e}_x - \mathrm{i}\mathbf{e}_y$. Compute for both cases the local intensity $I(z)$ and the local helicity $\mathbf{h}$:

$$I = |\mathcal{E}|^2\,, \qquad \mathbf{h} = \frac{\mathrm{Im}\,\mathcal{E}^* \times \mathcal{E}}{I} \tag{3.10}$$