Theoretische Physik III
- Quantenmechanik (SoSe 2019) -

Übungsblatt 06 ($20+\pi$ Punkte)[1]

Emission 13.05.19 – Absorption 21.05.16 – Digestion tba

Aufgaben mit Sternchen sind Klausurisomorph

▷ Aufgabe 1 (Angular Momentum Uncertainty Relations)* (3 Scores)

Please derive the expectation value and variances of the angular momentum x- and y-components in standard states $|\ell m\rangle$.

▷ Aufgabe 2 (Distribution of measurement values in hydrogen) (5 Punkte)

In the lecture you encountered the hydrogen electron ground state wave function (without spin), e Wellenfunktion des Grundzustandes eines Wasserstoffelektrons (ohne Spin) kennengelernt,

$$\psi_{1,0,0}(\vec{x}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}\,, \tag{1}$$

with a_0 Bohr-radius.

(a) Please derive the probability density for finding the electron, upon position measurement, at distance a from the nucleus.

(b) Show that the ground state wave function, in momentum representation ist given by

$$\tilde{\psi}(\vec{k}) = \frac{2^{3/2}}{\pi}\frac{1}{a_0^{5/2}}\frac{1}{\left(k^2+a_0^{-2}\right)^2}\,. \tag{2}$$

(c) What is the probability density for finding the the wavenumber $\vec{p}=\hbar\vec{k}$ in a measurement of the relative momentum $\vec{p}=\hbar\vec{k}$?

▷ Aufgabe 3 (Hydrogen – expectation values) (6 Punkte)

Please show that the expectation values of the electron-proton distance and average Coulombenergie of atomic hydrogen (neglecting spin) are related Zeigen Sie, daß die Erwartungswerte für den mittleren Abstand und die mittlere Coulombenergie im Wasserstoff durch

$$\langle\hat{r}\rangle_{nlm} = \left[3n^2 - l(l+1)\right]a_0/2\,, \qquad \left\langle\frac{e^2}{4\pi\varepsilon_0\hat{r}}\right\rangle_{nlm} = \frac{e^2}{n^2 a_0} \tag{3}$$

with a_0 the Bohr radius.

▷ Aufgabe 4 (Selection rules)* (6 Punkte)

[1] Aufgaben mit transzendenter Punktezahl sind fakultative Nüsse. Nüsse sind bekanntlich nahrhaft ...

The interaction of an atom with the electromagnetic field, in the dipole approximation, is described by a Hamiltonian piece

$$\hat{H}_{\text{int}} = -\vec{E} \cdot \hat{\vec{D}} \tag{4}$$

with $\hat{\vec{D}}$ the vector-operator "dipole moment", in case of atomic hydrogen $\hat{\vec{D}} = -e\hat{\vec{q}}$.

For the case of atomic hydrogen (neglecting spin), lease compute the matrix elements $\langle nlm|\hat{H}_{\text{int}}|n'l'm'\rangle$, and convince yourself of the so called *selection rules*

$$\Delta l \equiv l - l' = \pm 1\,, \qquad \Delta m \equiv m - m' = 0, \pm 1\,. \tag{5}$$

Selection rules play a prominent role in the light-matter interaction. From the rules above you may want to read off an hypothesis adressing the spin of a photon.

▷ **Aufgabe 5 ($\hbar$ in the lab . . .)** (π Punkte)

Suppose you are running a double slit experiment demonstrating the quantum mechanical wave aspect of matter. Some preliminary test run with monochromatic particles display an interference pattern with fringe distance a. You let the experiment run over night, you go home to have some sleep. Next morning you read in the newspaper that "somebody has changed the value of $\hbar$ over night, leaving all other natural constants untouched, however. Running to your lab, you come to the conviction, that with a changing $\hbar$, the fringe distance must have changed. "After all" you argue "the De-Broglie relation $\lambda = 2\pi\hbar/p$ implies a linear dependency of the wavelength, and thus the fringe distance, on $\hbar$. Just before entering the lab, you have some doubts. Certainty can only come from the measurement values – and these read WHAT?

<u>Remark:</u> Keep in mind, that in changing the value of $\hbar$, all kinds of things/relations may undergo some changes. The "size of an atom", for example (relativ to what?) The only proposition with unchanged trthvalue are propositions of the type "there are 17 potatoes in this box".

You may want to confront other physicists with our little problem. For example collegues from the experimental disciplines . . .