Theoretische Physik III
- Quantenmechanik (SoSe 2019) -

Übungsblatt 11 ($10 + \pi$ Punkte)[1]

Ausgabe 17.06.19 – Abgabe 25.06.19 – Besprechung n.V.

Aufgaben mit Sternchen sind Klausurisomorph

▷ **Aufgabe 1 (Harmonic oscillator in Heisenberg picture)** (8 Scores)

[“Pflicht” und klausurrelevant . . .]

We consider the harmonic oscillator which is placed in a homogeneous force field. THe Hamiltonian reads

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}q^2 - Fq \tag{1}$$

with F a real constant.

(a) Derive the classical equations of motion. Provide their general solutions.

(b) Quantise the system. Derive the Heisenberg equations of motion, and give their general solution.

Hint: It may prove useful to complete the square, $\frac{m\omega^2}{2}q^2 - Fq = \frac{m\omega^2}{2}\left(q - \frac{F}{m\omega^2}\right)^2 - \frac{F^2}{2m\omega^2}$.

▷ **Aufgabe 2 (Zwei-Niveau Atom im Lichtfeld)** (12 Scores)

The “two state system”, the “two level atom”, “spin in magentic field”, or “qubit” is characterized by a two-dimensional Hilbert space with basis $|e\rangle, |g\rangle$ (context: atomic physics), and a Hamiltonian – in rotating wave approximation (RWA) –

$$\hat{H}(t) = \hbar\omega_0\hat{\sigma}^\dagger\hat{\sigma} + \frac{\hbar\Omega_0}{2}e^{i\omega t}\hat{\sigma} + \frac{\hbar\Omega_0^*}{2}e^{i\omega t}\hat{\sigma}^\dagger \tag{2}$$

with $\hat{\sigma} = |g\rangle\langle e|$ and $\hat{\sigma}^\dagger = |e\rangle\langle g|$ transition operators.

(a) I claim that the dynamics of $\hat{H}$ has been the subject of some earlier lecture. Do you recall in what context?

(b) What are the Heisenberg equations of motion of the operators $\hat{\sigma}$, $\hat{\sigma}^\dagger$?

(c) What is the physical meaning of the expectation values of $\hat{\sigma}+\hat{\sigma}^\dagger$ and $\hat{\sigma}^\dagger\hat{\sigma}$, respectively?

(d) The explicit time dependence of $\hat{H}(t)$ is somewhat annoying. In order to cope with that nuisance, an interaction picture with respect to $\hat{H}_0 := \hbar\omega\hat{\sigma}^\dagger\hat{\sigma}$ appears recommendable. How does $\hat{H}(t)$ transform to the interaction picture? Would you agree that in the interaction picture the Schrödinger eqution for the transformed state vector $|\tilde{\psi}(t)\rangle := e^{\frac{i}{\hbar}\hat{H}_0 t)}|\psi(t)\rangle$ reads $i\hbar\frac{\partial}{\partial t}|\tilde{\psi}\rangle = \tilde{H}|\tilde{\psi}\rangle$ with

$$\tilde{H} = \hbar(\omega_0 - \omega) + \frac{\hbar\Omega_0}{2}\hat{\sigma} + \frac{\hbar\Omega_0^*}{2}\hat{\sigma}^\dagger \tag{3}$$

[1] Aufgaben mit transzendenter Punktezahl sind fakultative Nüsse. Nüsse sind bekanntlich nahrhaft . . .

(e) For an atom which is in the ground sate initially, please determine the probability to find the atom in the excited state at time t.

(f) Zum Hamiltonoperator $\tilde{H}$ kann man natürlich auch wieder die entsprechenden Heisenbergschen Bewegungsgleichungen aufstellen – und sogar lösen! Wir bitten darum ...

▷ Aufgabe 3 (Zenos Paradox) (π Scores)

Zeno of Elea, a presocratic, is well known for his paradoxes. One of his famous paradoxes is the "arrow paradox". To quote Wikipedia: In the arrow paradox, Zeno states that for motion to occur, an object must change the position which it occupies. He gives an example of an arrow in flight. He states that in any one (duration-less) instant of time, the arrow is neither moving to where it is, nor to where it is not. It cannot move to where it is not, because no time elapses for it to move there; it cannot move to where it is, because it is already there. In other words, at every instant of time there is no motion occurring. If everything is motionless at every instant, and time is entirely composed of instants, then motion is impossible.

In the quantum mechanics reformulation the quantum Zeno paradox states, that any attempt to monitor the dynamics of a quantum mechanical system leads to a complete freeze of the quantum mechnical state.

For illustration we take a resonantly driven two-level system from the previous problem. At time $t = 0$, the system is prepared in the ground state $|g\rangle$. At time t the state is measured with respect to the alternative $|e\rangle$ or $|g\rangle$. Und the provisio that between time $t = 0$ and time t no measurement took place, the atom will be found in the ground state with probability $P_g(t) = \cos(\Omega_0 t)$, and – concomitantly – in the excited state $P_e = 1 - \cos(\Omega_0 t)$.

The question is what happens, if state measurements are performed at times $n\Delta t$, $n = 1, 2, \ldots, N$ with $\Delta t = t/N$, in the limit $N \to \infty$? Confirm, that in this case the atom remains frozen in the ground state forever.

The quantum Zeno paradox – alternatively "A watched pot never boils" – is by no means a paradox, but reality indeed. Have a look at "Quantum Zeno effect", W. M. Itano et al., Phys. Rev A**41**, 2295 (1990); "Comment on 'Quantum Zeno effect' " von L. E. Ballentine, Phys. Rev. A**43**, 5165 (1991); "Reply to 'Comment on Quantum Zeno effect' ' " von W. M. Itano et al, Phys. rev. A**43**, 5168 (1991); and finally "Quantum Zeno effect without collapse of the wave packet" von V. Frerichs und A. Schenzle, Phys. Rev. A**44**, 1962 (1991).

Hinweis: Wenn Sie mit der Aufgabe fertig sind: nutzen Sie doch mal irgendeine Suchmaschine, Stichwort "Zeno Paradox", oder gehen gleich auf Wikipedia. Besser noch: gehen Sie auf die Uni-Seite der Bibliothek, dort auf e-journals, natürlich Physik, klicken sich auf Physical Review A durch, und laden einfach herunter: "Quantum Zeno effect", W. M. Itano et al., Phys. Rev A**41**, 2295 (1990). Da lernen Sie dann, wie man ein "permanten-nachgucken-Instrument" im Labor realisieren kann. Anschließend erfreuen Sie sich an einer Debatte über die Grundlagen der Quantenmechanik: "Comment on 'Quantum Zeno effect' " von L. E. Ballentine, Phys. Rev. A**43**, 5165 (1991); "Reply to 'Comment on Quantum Zeno effect' ' " von W. M. Itano et al, Phys. rev. A**43**, 5168 (1991); "Quantum Zeno effect without collapse of the wave packet" von V. Frerichs und A. Schenzle, Phys. Rev. A**44**, 1962 (1991).