Einführung in die Quantenoptik II

Sommersemester 2019

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Übungsaufgaben Blatt 2

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Aufgabe 2.1 – Laser feeling (10 Punkte)

Please look up the typical elements of a laser: a cavity for the laser mode, an active medium, a mechanism for pumping the medium, an optical system to shape the laser beam.

Look up typical numbers for a laser:

(1) the product ('quality factor' Q) between the resonance frequency ω_c and the 'photon lifetime' τ in the cavity. In the lecture, we shall use $\kappa = 1/\tau$ as 'cavity decay rate'.

(2) the ratio between the input power (needed for pumping the active medium) and the optical output power. This 'efficiency' η is often not very large – this is one of the reasons why laser-induced fusion is probably not a very efficient way of producing energy, for example.

(3) the product of cavity lifetime τ and the flux of photons (photons per second) in the laser beam. We shall see that this is a good estimate for the photon number $\langle n \rangle$ in the cavity. Try to find an estimate. For a 'microlaser' or 'micro maser' (experiments by Nobel prize winner Serge Haroche in Paris), this number is small.

(4) try to find from a 'laser catalogue' information about the fluctuations (stability) of the laser beam power. What physical quantities are used to describe this? We shall be interested in the lecture in the standard deviation Δn of the photon number inside the laser cavity: think how one could translate the catalogue information into the number Δn.

Aufgabe 2.2 – Semiclassical photon gain (10 Punkte)

A simplified model for the (average) photon number N in a laser is given by the equation of motion

$$\frac{dN}{dt} = \frac{G_0 N}{1 + \beta N} - \kappa N \tag{2.1}$$

where κ is a loss rate (photons leave the laser cavity). The rate G_0 is called the 'small-signal amplification' (why?) and β the 'saturation parameter' (why?).

(1) This equation is nonlinear and therefore difficult to solve. Take your favorite numerical environment and solve the equation numerically. Play with initial data and the parameters. The 'marginal case' $G_0 = \kappa$ is particularly interesting because the solution may take longer time to equilibrate.

Give arguments that qualitatively the behaviour of the non-linear equation *below threshold* is similar to the linear case: the photon number will decay to zero, whatever its initial value.

(2) Make the weak-saturation approximation, $\beta N \ll 1$ and solve the linearized equation. Explain why the laser threshold is given by the condition 'losses = gain', $\kappa = G_0$.

(3) By evaluating the saturated gain term for larger N, you can qualitatively sketch the solution for the photon dynamics *above threshold*. Argue that a stationary state is reached where $N(t) \to N_\infty = (G_0 - \kappa)/\beta$ and that this state is reached 'from below' $N(t) < N_\infty$ for all t, provided $N(0) < N_\infty$.

(4) Perform a stability analysis of the stationary state (above threshold) and determine the time scale on which $N = N_\infty$ is reached. (Guess: $1/\kappa$.)