Einführung in die Quantenoptik II

Sommersemester 2019

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Übungsaufgaben Blatt 6 Ausgabe: 12. Juni 2019

Diskussion: 21. Juni 2019

Aufgabe 6.1 – Resonance Fluorescence Technicalities (10 Punkte)

We have sketched in the lecture how the correlation function $C_\sigma(\tau) = \langle\sigma^\dagger(t+\tau)\,\sigma(t)\rangle$ can be computed for $t \to \infty$ with the quantum regression formula. In this context, we have introduced the skew density operator $P(\tau|\sigma\rho_{\rm st})$ that evolves according to the master equation and from the initial condition $P(0|\sigma\rho_{\rm st}) = \sigma\rho_{\rm st}$.

(1) Show that $P(0)$ has the trace $(s_1 - {\rm i}s_2)/2$ and is described by the complex Bloch vector **S** with components (why is **S** complex?)

$$S_1 = \frac{1+s_3}{2}\,, \qquad S_2 = \frac{1+s_3}{2{\rm i}}\,, \qquad S_3 = \frac{-s_1 + {\rm i}s_2}{2} \tag{6.1}$$

where s $= (s_1, s_2, s_3)$ is the steady-state Bloch vector corresponding to $\rho_{\rm st}$.

(2) The time evolution of the Bloch vector is given by the system of equations

$$\frac{{\rm d}S_1}{{\rm d}\tau} = -\Gamma S_1 + \Delta S_2 \qquad \frac{{\rm d}S_2}{{\rm d}\tau} = -\Delta S_1 - \Gamma S_2 - \Omega S_3$$
$$\frac{{\rm d}S_3}{{\rm d}\tau} = \Omega S_2 - \gamma(S_3 + {\rm tr}\,P) \tag{6.2}$$

where Δ is the detuning, Ω the (real) Rabi frequency and Γ and γ the decay rates for coherences and populations. Compute the stationary solution of these equations and check that in the stationary state $P_{\rm st}$, the expectation value of $\sigma^\dagger = (\sigma_1 + {\rm i}\sigma_2)/2$ is given by

$${\rm tr}(\sigma^\dagger P_{\rm st}) = |{\rm tr}(\sigma\rho_{\rm st})|^2 \tag{6.3}$$

(3) Eqs.(6.2) can be written as $\dot{\rm S} = -\mathbf{B}\mathrm{S} + \ldots$ with a 3×3 matrix $\mathbf{B}$. Show that the characteristic equation for its eigenvalues can be written as

$$(\Gamma-\lambda)^2(\gamma-\lambda) + \Omega^2(\Gamma-\lambda) + \Delta^2(\gamma-\lambda) = 0 \tag{6.4}$$

Solve this equation approximately in the two cases $|\Omega| \ll \gamma/2 < \Gamma$ and $|\Omega| \gg \Gamma$. Write a few sentences on the physical interpretation for the real and imaginary parts of the eigenvalues λ.

Aufgabe 6.2 – Anti-bunching (10 Punkte)

The correlations of the fluorescence intensity I is related to the probability of detecting two photons, one at t and another one at the later time t'. According to Glauber's photodetector theory, the joint detection rate is proportional to the correlation function (time and normal order)

$$C_I(t'-t) = \langle \sigma^\dagger(t)\sigma^\dagger(t')\sigma(t')\sigma(t)\rangle \tag{6.5}$$

(1) Construct with the regression approach a formula that gives this quantity in terms of a conditional density operator $P(t'-t|\ldots)$. Justify the wording "just after the time t, the two-level system is for sure in its ground state" and check that the initial state for $P(t'-t|\ldots)$ is (nearly) an ordinary density operator.

(2) Show that for $t'=t$, $C_I(0)=0$ and give an interpretation for this result.

(3) In a classical description, the fluorescence intensity $I(t)$ is proportional to $|d(t)|^2$ where d is a complex dipole amplitude that is represented in the quantum theory by the operator σ. Show with the Schwarz inequality that in this picture, the correlation function satisfies

$$\langle |d(t)|^4\rangle \geq \langle |d(t)|^2 |d(t')|^2\rangle \quad \text{and} \quad \langle |d(t)|^4\rangle \geq \langle |d(t)|^2\rangle^2 \tag{6.6}$$

where the averages are taken with respect to a probability distribution for $d(t')$. For this reason, the quantum result $C_I(0)=0$ is called "non-classical": it cannot be reproduced by replacing quantum operators by classical variables with whatever distribution function.