Introduction to Quantum Optics

Winter term 2006/07
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Problem set 7 Hand out: 11 Jan 2007
Hand in: 25 Jan 2007

Problem 7.1 – Two-level photodetector (6+4* points)

In the lecture, we have derived the equation for the spin operators σ, σ_3 of a two-level atom coupled to the quantized light field.

(i) Write down a formal solution for $\sigma(t)$ in terms of the initial value $\sigma(0)$ and the history of the field $\mathcal{E}(t')$ and insert this into the equation for $\mathrm{d}\sigma_3/\mathrm{d}t$.

(ii) Take the expectation value of $\mathrm{d}\sigma_3/\mathrm{d}t$ with as initial state the atom in the ground state and the field in an arbitrary state. This operator gives the average rate of exciting the atom into the excited state. Work out this rate assuming that the relaxation rate γ of the excited state is very large (model of a 'fast photodetector').

(iii*) Try to express, in the general case, the average excitation rate in terms of the ('instantaneous') spectrum $S(\omega; t)$ of the photon field and a spectral response of the two-level detector.

The equations we found are

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma = -\left[\mathrm{i}\omega'_{eg} + \tfrac{1}{2}\gamma\right]\sigma + \mathrm{i}\sigma_3\mathcal{E}_0(t) \tag{7.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma_3 = 2\mathrm{i}\sigma^\dagger\mathcal{E}_0 - 2\mathrm{i}\mathcal{E}_0^\dagger\sigma - \gamma\sigma^\dagger\sigma \tag{7.2}$$

where ω'_{eg} is the shifted transition frequency, γ is the excited state decay rate, and $\mathcal{E}_0(t)$ is the freely evolving electric field, expressed in units of frequency.

Problem 7.2 – Optical lattices (7 points)

You have seen the dipole potential in the lecture; it is for a two-level atom at sufficiently large detuning, proportional to the light intensity. Laser beams that are crossed create a spatial interference pattern where this potential is modulated on the scale of the laser wavelength. One gets a lattice of potential minima that can be controlled by the light intensity, polarization, and the angles of the laser beams. Such 'optical lattices' are employed in current experiments to simulate solid state system with ultracold atoms and Bose-Einstein condensates; massively parallel quantum computers implemented with atoms trapped in the lattice are also considered.

(i) As a first glimpse, estimate the depth of the dipole potential for a pair of laser beams with a detuning of a few nm (in units of wavelength). Take the dipole matrix elements of hydrogen to fix numbers. A few Watt of laser power are easily available.

(ii) Consider three plane waves that propagate in the xy-plane, forming for example a 'Mercedes star'. What spatial pattern do you get for the light intensity? Consider separately the cases of a polarization along the z-axis and in the $xy-$plane. (The polarization vector must be orthogonal to the wave vector, of course.) How do the angles between the laser beams change the intensity pattern? What about the relative phase between the beams (they are assumed to have all the same frequency).

Fee free to play with some programme to make pictures of the potential.

Problem 7.3 – Spontaneous emission and photon recoil (7 points)

We have seen that for each spontaneous emission of a photon, the momentum of the atom changes by $\hbar\mathbf{k}$ where $-\mathbf{k}$ is the k-vector of the emitted photon. After a large number of emission events, these recoils lead to a 'random walk' of the atomic momentum. Their directions are random and distributed according to the emission pattern of a radiating dipole. This process leads to a fundamental lower limit for the temperature of laser cooling.

(i) Work out the averages $\hbar^2\overline{k_i k_j}$ of the recoil momentum if both dipole and laser field are linearly polarized along the z-axis, say. One of the results is $\overline{k_x^2} = \frac{2}{5}(\omega/c)^2$ for example. What about the other components?

(ii) The discussion of the 'random walk' due to photon recoil is very simple if subsequent emission processes are independent, i.e., if the correlation time τ_c is very short. Take as an estimate of τ_c the inverse photon emission rate, $1/(\gamma\rho_{\rm ee})$ which can be compared to the 'damping rate' α of the atomic velocity provided by Doppler cooling (see lecture). What is the maximum value of $\alpha\tau_c$ (as a function of detuning and laser power)? What are typical values of this number for a few alkali-like atoms, one 'light', another one 'heavy'? You can restrict the estimate to the 'D lines' of the alkali spectra.