Chapter 4

Laser cooling

(2–3 lectures)

In this chapter, we give an introduction to the "mechanical effects of light" on atoms, i.e., how the motion of atoms can be manipulated with laser light. This field has given rise to various techniques of laser cooling and atom trapping. It has opened the way to reach extremely low temperatures (in the nanoKelvin range) and, ultimately, the Bose-Einstein condensation of low-density gases. Recent Nobel prizes have been awarded to key scientists in this enterprise (1997: laser cooling, 2001: Bose-Einstein condensation).

4.1 Light forces

4.1.1 Electric dipole

Our working horse is still the electric dipole moment of an atom that mediates the coupling to the electric field of a laser. From the electrostatics lecture, you know that a permanent dipole in an electric field is subject to the potential energy

$$V_{\rm es} = -\mathbf{d}\cdot\mathbf{E},$$

and you get a force by taking the gradient of this quantity:

$$\mathbf{F}_{\rm es} = \sum_{i=3}^{3} d_i \nabla E_i.$$

This result is basically true also for the induced dipole of an atom or molecule. However, the calculation is a little bit more involved (i) because the dipole is induced (should one add a gradient of $\mathbf{d}$? ... no) and (ii) the dipole is oscillating. The force thus shows oscillations at the laser frequency which are very rapid compared to the timescale on which (thermal or slow) atoms move. A more useful quantity is thus the "force averaged over one optical period". You are invited in the exercises to study this model in more detail. For a classical dipole that oscillates at the frequency of a monochromatic laser field, one gets contributions from both the electric (Coulomb) and magnetic (Lorentz) forces. Taking the average, these combine to give

$$\mathbf{F}_{\rm lf} = \sum_{i=3}^{3} d_i^* \nabla E_i(\mathbf{r}) + \text{c.c.}$$

where now $\mathbf{d}$ and $\mathbf{E}$ are the positive frequency amplitudes of the dipole and field. (We recall our convention that $\mathbf{E}(t) = \mathbf{E}\,\mathrm{e}^{-\mathrm{i}\omega_L t} + \mathbf{E}^*\,\mathrm{e}^{\mathrm{i}\omega_L t}$.)

In the exercises, you are invited to analyze the light force for a classical electric dipole. Assuming that the dipole responds linearly to the field (with a polarizability $\alpha(\omega_L)$ dependent on the laser frequency), you will find contributions proportional to the absorption ($\mathrm{Im}\,\alpha$) and the dispersion ($\mathrm{Re}\,\alpha$). A similar distinction can be made for the more quantum-mechanical two-level atom that we discuss now.

4.1.2 Two-level atom

The dipole now becomes an operator (we still treat the light field classically), and we have to include an additional average over the quantum state of the two-level atom. This state will in general be a mixed one, and the corresponding density matrix element is

$$\langle\mathbf{d}\rangle = \mathbf{D}\,\rho_{\rm eg}$$

where $\mathbf{D}$ is the c-vector of dipole matrix elements and $\rho_{\rm eg}$ is the optical coherence in the rotating frame. The average light force is now given by

$$\langle\mathbf{F}_{\rm lf}\rangle = \sum_i \left(\rho_{\rm eg}^* D_i^* \nabla E_i(\mathbf{r}) + \text{c.c.}\right) = -\hbar\,\mathrm{Re}\left(\rho_{\rm eg}^* \nabla\Omega(\mathbf{r})\right) \qquad (4.1)$$

where in the last step we have introduced the Rabi frequency (and the c-vector $\mathbf{D}$, being constant, has been included under the gradient).

When a laser field of a typical duration (longer than the natural lifetime $1/\gamma$ of the excited state) is applied on the two-level atom, the atomic density matrix can safely be evaluated in the steady state. We recall that in this limit, the atomic coherence and the excited state population are given by

$$\rho_{\rm eg} = \frac{\Omega}{2}\frac{\Delta - {\rm i}\gamma/2}{\Delta^2 + \gamma^2/4 + |\Omega|^2/2} \quad (4.2)$$

$$\rho_{\rm ee} = \frac{|\Omega|^2/4}{\Delta^2 + \gamma^2/4 + |\Omega|^2/2} \quad (4.3)$$

where Δ is the detuning of the laser field and γ the decay rate of the excited state. In the following, we discuss two contributions to the force (4.1) related to imaginary and real part of this optical coherence. We see here that the steady state $\rho_{\rm eg}$ is position-dependent. Since the force (4.1) only involves the gradient of the Rabi frequency, it is clear that in general the force cannot be written as the gradient of a potential.

4.1.3 Radiation pressure force

We can always write the gradient of the Rabi frequency in (4.1) in terms of an amplitude and a phase gradient:

$$\nabla\Omega = \frac{\Omega}{|\Omega|}\nabla|\Omega| + {\rm i}\nabla\phi\,\Omega$$

where ϕ is the phase of Ω. These two gradients give two contributions to the light force:

$$\langle \mathbf{F}\rangle = \mathbf{F}_{\rm rp} + \mathbf{F}_{\rm dip} \quad (4.4)$$

$$\mathbf{F}_{\rm rp} = \hbar\gamma\,\nabla\phi\frac{|\Omega|^2/4}{\Delta^2 + \gamma^2/4 + |\Omega|^2/2} \quad (4.5)$$

$$\mathbf{F}_{\rm dip} = -\frac{\hbar\Delta}{4}\frac{\nabla|\Omega|^2}{\Delta^2 + \gamma^2/4 + |\Omega|^2/2} \quad (4.6)$$

To give an interpretation of these two contributions, let us first consider a plane wave laser field. The Rabi frequency then has constant modulus, and only a phase gradient occurs. The light force is then given by

$$\mathbf{F}_{\rm rp} = \hbar\mathbf{k}\,\gamma\frac{|\Omega|^2/4}{\Delta^2 + \gamma^2/4 + |\Omega|^2/2} = \hbar\mathbf{k}\,\gamma\,\rho_{\rm ee}. \quad (4.7)$$

We see that the force is oriented parallel to the wave vector of the laser field or parallel to the "photon momentum" $\hbar\mathbf{k}$. The atom is thus "pushed forward" by the laser beam — this force is called "radiation pressure". With the photon picture, we can give an intuitive interpretation of the radiation pressure: it is given by the photon momentum times the number of times per unit time, $\gamma\rho_{\rm ee}$ that the atom decays spontaneously from its excited state. This number occurs because each spontaneous emission has been preceded by the absorption of a photon from the laser field, giving the atom a momentum $\hbar\mathbf{k}$. If the photon emission is stimulated, back into the laser beam, the atom also gives back the absorbed momentum. But if the emission is spontaneous, the photon is emitted in a random direction (with a probability given by the dipole radiation pattern), and there is no net momentum transfer on average.

Let us make an order of magnitude estimate for the radiation pressure force. The corresponding acceleration is maximally

$$a_{\rm rp} \leq \frac{\hbar k}{2m}\gamma \sim \frac{10^{-34}\,{\rm J\,s}}{10^{-26}\,{\rm kg}}\frac{10^{7}\,{\rm s}^{-1}}{10^{-7}\,{\rm m}} \sim 10^{8}\,{\rm m/s}^2$$

— this is really large: ten millions times the gravitational acceleration.

With this force, one can decelerate an atom very fast: if we start with an atomic beam at a typical thermal velocity (for a few hundred K), $v \sim 10^3\,{\rm m/s}$, then the beam should come to rest on a timescale

$$\frac{v}{a_{\rm rp}} \sim \frac{10^3\,{\rm m/s}}{10^8\,{\rm m/s}^2} = 10^{-5}\,{\rm s},$$

which corresponds to a distance of about $v^2/a_{\rm rp} \sim 1\,{\rm cm}$. But this estimate neglects the Doppler effect: one has to adjust the laser frequency to stay in resonance with the atomic transition. If this is not done, the atom gets out of resonance after a velocity change Δv such that the Doppler shift $k\Delta v \sim \gamma$. This gives

$$\Delta v \sim \frac{\gamma}{k} \sim \frac{10^7\,{\rm s}^{-1}}{10^7\,{\rm m}} = 1\,{\rm m/s}$$

which is not very much compared to a thermal velocity. In practice, there are two options to keep in resonance: either the laser frequency is "chirped"[1]and is getting smaller while the atoms slow down (smaller

[1]to chirp: *zwitschern*.

Doppler shift). Or one uses the Zeeman effect in a magnetic field that varies in space. The atomic beam flows through a tube where the field is initially strong to compensate the large Doppler shift and gets gradually weaker when the atoms get slower.

The Doppler effect also plays a crucial role to cool atoms. This we shall see in the next section.

4.1.4 Dipole force

What about the second contribution $\mathbf{F}_{\rm dip}$ in (4.6)? It requires an intensity gradient, and this may remind us of the shift of the two atomic "dressed" levels with respect to the "bare" transition frequency. Let us focus to simplify things on the limit of a large detuning, $\Delta \gg |\Omega|, \gamma$.

In that limit, we can neglect spontaneous emission and use the formulas of the previous term for the dressed levels: $\Delta E_{g,e} = \pm\frac{1}{2}\hbar\Delta\sqrt{1+|\Omega|^2/\Delta^2}$ for the ground and excited states. Taking the negative gradient, these energy shifts give a force

$$\mathbf{F}_{\rm g,e} = \mp\frac{\hbar}{4\Delta}\nabla|\Omega|^2$$

Assuming now that the atom, moving not too fast in the laser field, follows the dressed level connected to the ground state, we get the large-detuning limit of (4.6). The dipole force can thus be understood from the spatially varying eigenenergies of the atom-field interaction Hamiltonian.

A more careful analysis shows that even for $\Delta \sim \Omega \sim \gamma$, one may still use the idea of a gradient of level shifts, averaged over the atomic populations:

$$\langle\mathbf{F}\rangle = -\sum_{\alpha=1,2}\nabla E_\alpha \rho_{\alpha\alpha} \tag{4.8}$$

But the transitions that occur between these levels due to spontaneous emission have to be calculated more carefully, since both are a mixture of ground and excited states.

To conclude, let us mention that the dipole force derives from a potential

$$\begin{aligned} U_{\rm dip}(\mathbf{r}) &= \frac{\hbar\Delta}{2}\log\Big(\Delta^2+\gamma^2/4+|\Omega(\mathbf{r})|^2/2\Big) \\ &= \frac{\hbar\Delta}{2}\log\left(1+\frac{\gamma^2/4}{\Delta^2+\gamma^2/4}\frac{I(\mathbf{r})}{I_{\rm sat}}\right) + \text{const.} \end{aligned}$$

In the second line, we have used the "saturation intensity" that is characteristic for the atomic transition used (it is related to the linewidth γ and the transition wavelength). It often happens that one works in the limit of large detuning (to enhance the dipole force relative to radiation pressure). In this regime, the dipole potential can be written

$$U_{\rm dip}(\mathbf{r}) \approx \frac{\hbar|\Omega(\mathbf{r})|^2}{4\Delta} = \frac{\hbar\gamma^2}{8\Delta}\frac{I(\mathbf{r})}{I_{\rm sat}}$$

which is proportional to the light intensity.

Note that the dipole potential is attractive towards the regions of large intensity when the detuning is negative ("red detuning"). In this way, atoms can be trapped in the focus of a far-detuned laser beam ("optical tweezer"). Another simple dipole trap are the nodes of a standing laser wave with "blue" (= positive) detuning. This trap has the advantage that at the bottom of the potential well, the atoms are "in the dark" (the light intensity is zero at the nodes), and spontaneous emission is suppressed. Finally, with an "evanescent" wave that shows an intensity gradient perpendicular to a glass surface (on its back side, a laser beam is totally reflected), one can realize one-dimensional attractive or repulsive potentials — the repulsive potential being an interesting candidate for an "atom mirror" if the atoms arrive with a not too large kinetic energy.

4.2 Doppler cooling

This is the code word for a simple cooling mechanism that works with two-level atoms.

We have seen that the radiation pressure force can exert a very large acceleration on an atom provided the laser field is kept on resonance. This can be used to slow down atoms, but it is not yet cooling.

4.2.1 Light friction: "optical molasses"

Cooling means that we also want to reduce the velocity spread of an atomic ensemble. This can be done, at least in one dimension, in a configuration with two laser beams, impinging from the left and right onto the atom.

We have to take into account the Doppler shift of the laser frequency when the atom is moving. In a single plane wave beam with wave vector $\mathbf{k}$, the Doppler shift leads to a velocity-dependent detuning:

$$\omega_L \mapsto \omega_L - \mathbf{k}\cdot\mathbf{v} \Longrightarrow \Delta \mapsto \Delta - \mathbf{k}\cdot\mathbf{v}$$

The laser frequency increases in the frame of the moving atom when atomic velocity and laser wave vector are opposite — remember the tune of the fire department car.

We thus get a velocity-dependent radiation pressure force

$$\mathbf{F}_{\rm rp}(\mathbf{v}) = \hbar\mathbf{k}\,\gamma\frac{\Omega^2/4}{(\Delta-\mathbf{k}\cdot\mathbf{v})^2+\gamma^2/4+|\Omega|^2/2}.$$

In the following, we put $\gamma'^2/4 = \gamma^2/4 + |\Omega|^2/2$.

If we have two counter-propagating beams with wave vectors $\pm\mathbf{k}$ and the same power and if we are allowed to sum their radiation pressure forces, we get the following total force

$$\mathbf{F}_{\rm Dc}(\mathbf{v}) = \frac{\hbar\mathbf{k}\,\gamma\Omega^2}{4}\left(\frac{1}{(\Delta-\mathbf{k}\cdot\mathbf{v})^2+\gamma'^2/4} - \frac{1}{(\Delta+\mathbf{k}\cdot\mathbf{v}^2+\gamma'^2/4}\right).$$

This function shows a positive peak for velocities with $\mathbf{k}\cdot\mathbf{v} = \Delta$ (radiation pressure of the beam coming from the left) and a negative peak for $\mathbf{k}\cdot\mathbf{v} = -\Delta$ (beam from the right). If we choose a detuning $\Delta < 0$, we get a force that is opposite to the direction of motion in the interval $\Delta < \mathbf{k}\cdot\mathbf{v} < -\Delta$. The atom moves to the right and the beam coming from the right gets closer to resonance, giving a larger radiation pressure. The atom moves to the left, and the beam from the left comes into resonance. This situation is called an "optical molasses"[2] because the atomic velocity gets rapidly damped to zero once it lies in the interval mentioned above.

In particular, around zero velocity we get

$$\mathbf{F}_{\rm Dc}(\mathbf{v}) \approx \hbar\mathbf{k}\,\Omega^2(\mathbf{k}\cdot\mathbf{v})\frac{\Delta\gamma}{(\Delta^2+\gamma'^2/4)^2}.$$

In a one-dimensional problem, we thus have a "friction force"

$$\begin{aligned} F_{\rm Dc} &= -m\alpha v \\ \text{with } \alpha &= -\frac{\hbar k^2}{m}\Omega^2\frac{\Delta\gamma}{(\Delta^2+\gamma'^2/4)^2} \end{aligned} \qquad (4.9)$$

[2] molasses: *Sirup, Abfallprodukt bei der Herstellung von Rum*

This friction allows to cool the atom: the atomic kinetic energy decays like

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{mv^2}{2} = -m\alpha v^2.$$

The maximum value of the friction coefficient is obtained for a detuning $\Delta \sim -\gamma$ (up to a numerical factor that you are invited to compute in the exercises).

4.2.2 Limit of Doppler cooling

The equilibrium temperature in optical molasses ("Doppler limit") is determined by the balance between friction (reduces the kinetic energy) and the increase of the atomic momentum due to the random recoil momenta occurring in spontaneous emission. These random recoils result in a random walk of the atomic momentum and a linear increase of the momentum variance as times goes on:

$$\langle\delta p^2(t)\rangle = 2D_p t$$

Here, D_p is called the momentum diffusion coefficient. If the average momentum of the atom is zero, we may also write this equation in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle p^2\rangle\Big|_{\rm diff} = 2D_p$$

The balance between friction and diffusion is therefore given by

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\langle p^2\rangle}{2m} = -\frac{\alpha}{m}\langle p^2\rangle + \frac{D_p}{m}$$

and we get an equilibrium kinetic energy ('temperature')

$$k_B T = \frac{\langle p^2\rangle}{2m} = \frac{D_p}{2m\alpha} \qquad (4.10)$$

The larger the friction coefficient, the lower the temperature.

A simple model for the momentum diffusion is based on the random photon momenta that the atom exchanges with the spontaneously emitted photons. The emission processes occur with a rate $\Gamma = \gamma\,\rho_{\rm ee}$ such that $N = \Gamma\,\delta t$ emissions occur during a time δt.

The variance of the photon recoil momenta is (more precisely, we take the projection onto the direction of the cooling laser)

$$\begin{aligned}\langle \delta p^2 \rangle &= \Big\langle \Big(\sum_{\alpha=1}^{N} \hbar k_{x,\alpha} \Big)^2 \Big\rangle = N\hbar^2 \left\langle k_x^2 \right\rangle = \dots \\ &= \gamma \delta t \, \rho_{\mathrm{ee}} \frac{2}{5} \hbar^2 k^2\end{aligned}$$

We use the fact that each spontaneous photon is emitted independently of the others and for each one, the same distribution applies. In the last step, we have used the dipole radiation pattern as the probability distribution for the directions of the emitted photons. The dipole moment is assumed to be linearly polarized and orthogonal to the cooling laser (accurate for a linear laser polarization). Finally, the magnitude k of the spontaneous photon wavevector is nearly equal to that of the cooling laser (the difference is of the order of Δ/c, but this is very small compared to $k = \omega/c$).

We thus see that the momentum variance increases linearly with time δt and get a diffusion coefficient

$$D_p = \frac{\delta p^2}{\delta t} = \frac{1}{5} \gamma \hbar^2 k^2 \frac{\Omega^2/4}{\Delta^2 + \gamma'^2/4}$$

where we have used for ρ_{ee} the steady state value for an atom with $v = 0$.

Finally, the lowest temperature is given by

$$\begin{aligned}k_B T_D &= \min \frac{D_p}{2m\alpha} = \dots = \min \frac{-\hbar}{40} \frac{\Delta^2 + \gamma'^2/4}{\Delta} \\ &= \frac{\hbar}{40} \min \sqrt{\gamma^2 + 2\Omega^2} = \frac{\hbar\gamma}{40} \qquad (4.11)\end{aligned}$$

As an order of magnitude, $T_D \approx 10\,\mu$K for typical linewidths.

4.3 The magneto-optical trap

Combination of Doppler shift and Zeeman shift like for Zeeman slower.

Quadrupole magnetic field

$$\mathbf{B}(\mathbf{r}) = \begin{pmatrix} bx \\ by \\ -2bz \end{pmatrix}$$

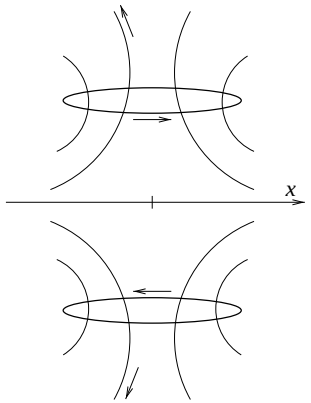

Figure 4.1: Quadrupole field lines produces by a pair of "anti-Helmholtz" coils.

generated with "anti-Helmholtz" coils (currents in opposite directions; sketch field lines). See figure 4.1.

Simplest model: 1D, $0 \leftrightarrow 1$ transition. Counterpropagating beams with $\sigma_+ \, \sigma_-$ polarizations.

Beam from left drives $0 \leftrightarrow +1$ transition with Zeeman shift βx (upper level shifted up). Radiation pressure force

$$F_L(v,x) = \frac{C}{(\Delta - kv - \beta x)^2 + \gamma'^2/4}$$

with constant $C = \hbar k \gamma \Omega^2/4$ and $\beta = \mu b/\hbar$.

Force for beam from the right: opposite sign, opposite Doppler shift, opposite Zeeman shift (for upper level -1):

$$F_R(v,x) = -\frac{C}{(\Delta + kv + \beta x)^2 + \gamma'^2/4}$$

Sum of radiation pressure forces for small velocities and short distances:

$$F_{MOT}(v,x) \approx 2C\,\Delta \frac{kv + \beta x}{(\Delta^2 + \gamma'^2/4)^2}$$

gives both friction force and restoring force for red detuning $\Delta < 0$. See figure 4.2.

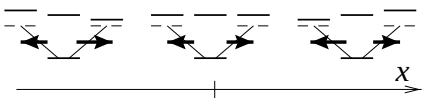

Figure 4.2: Trapping force in a MOT. Imbalance of radiation pressure forces due to the inhomogeneous magnetic field.

Estimate for typical oscillation frequency for atom at rest

$$\omega_x^2 \sim 10^4\,\mathrm{s}^{-2}\frac{\Omega^2}{\gamma^2}$$

hence in the 10 ms range. Much slower than damping rate from friction – "overdamped" regime.

Result: atoms trapped in spatial region where $|\Delta - \beta x| \leq \gamma$. Temperature at least at Doppler limit.

4.4 Sisyphus cooling

"Golden age" of laser cooling (end 1980's): atom beam experiments with transverse laser beams (W. Phillips, S. Chu). Goal: "transverse cooling", i.e. narrow the transverse velocity distribution. One had the theory of Doppler cooling and observed that ...

... the cooling was more efficient than expected, and a much lower temperature could be reached;

... the limiting temperature was not "universal" (like in Doppler cooling (4.11), but depended on the laser intensity and the detuning.

Good news for two reasons: (i) possible to reach lower temperatures, (ii) need to find "new cooling mechanism"

Explanation found at Ecole Normale Supérieure (Paris) by C. Cohen-Tannoudji and J. Dalibard. Essential ingredient: optical pumping in the Zeeman degenerate ground state.

Here: simple 1D model to understand basic features. Compute friction coefficient (larger than in Doppler cooling).

4.4.1 Model

Light field: standing wave with "lin⊥lin" polarization, i.e.

$$\mathbf{E}(z) = \mathcal{E}\left(\mathbf{e}_x\,\mathrm{e}^{\mathrm{i}kz} + \mathbf{e}_y\,\mathrm{e}^{-\mathrm{i}kz}\right)$$

Can be written as a sum of $\sigma^\pm$ polarizations

$$\mathbf{E}(z) = -\frac{\mathcal{E}}{\sqrt{2}}\left[\mathbf{e}_+\left(\mathrm{e}^{\mathrm{i}kz} - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}kz}\right) - \mathbf{e}_-\left(\mathrm{e}^{\mathrm{i}kz} + \mathrm{i}\,\mathrm{e}^{-\mathrm{i}kz}\right)\right] \tag{4.12}$$

Atom: $1/2 \leftrightarrow 3/2$ transition. Simple model including a Zeeman degeneracy in the ground state (essential for Sisyphus cooling). Work in limit where the atom is essentially in the ground state (large detuning, weak laser intensity).

4.4.2 Ground state light shifts

Compute effective Hamiltonian for the ground state. For a $1/2 \leftrightarrow 1/2$ transition, we have seen in the exercises

$$H_{\mathrm{eff}}(1/2 \leftrightarrow 1/2) = \frac{D^2}{\hbar\Delta}\left(\frac{1}{3}|\mathbf{E}|^2 - \frac{1}{3}\boldsymbol{\sigma}\cdot\underbrace{\mathrm{Im}\,\mathbf{E}^*\times\mathbf{E}}_{\text{effective B field}}\right)$$

For our transition, one only has to change the coefficients (see exercises)

$$H_{\mathrm{eff}}(1/2 \leftrightarrow 3/2) = \frac{D^2}{\hbar\Delta}\left(\frac{2}{3}|\mathbf{E}|^2 + \frac{1}{3}\boldsymbol{\sigma}\cdot\mathrm{Im}\,\mathbf{E}^*\times\mathbf{E}\right)$$

Total field intensity in our case: $|\mathbf{E}(z)|^2 = 2|\mathcal{E}|^2$. Effective magnetic field

$$\mathrm{Im}\,\mathbf{E}^*\times\mathbf{E} = -2|\mathcal{E}|^2\,\mathbf{e}_z\,\sin 2kz$$

Hence the effective Hamiltonian is diagonal in the basis (with quantization axis along z, parallel to the laser beams):

$$\begin{aligned} H_{\mathrm{eff}}(z) &= \hbar\Delta'\Big(2 - \sigma_z\,\sin 2kz\Big) \\ \hbar\Delta' &= \frac{2D^2|\mathcal{E}|^2}{3\hbar\Delta} \end{aligned}$$

The eigenvalues of this Hamiltonian have sinusoidal modulations (see figure 4.3). The state $|-\rangle = |-1/2\rangle$ has its lowest energy (for detuning $\Delta < 0$)

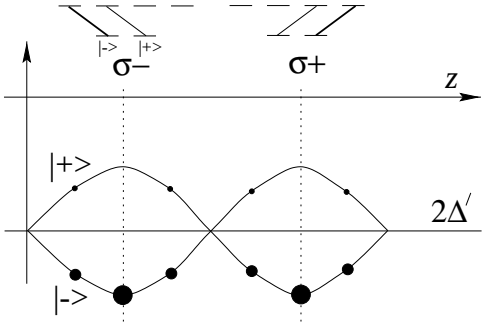

Figure 4.3: Light shift potentials and steady state populations for Sisyphus cooling.

at the positions $2kz = \pi/2$ where the helicity of the light points along $-\mathbf{e}_z$, and the light is purely σ^- polarized. This can be checked from (4.12) because $\mathrm{e}^{\mathrm{i}kz} = \sqrt{\mathrm{i}}$ there. One half-wavelength to the right, the light polarization is σ^+, and the state $|+\rangle = |+1/2\rangle$ has the lowest energy. This is because the transition $|+1/2\rangle \leftrightarrow |e, +3/2\rangle$ has a larger Clebsch-Gordan coefficient and hence a larger Rabi frequency.

4.4.3 Optical pumping rates

Need rate equations for optical pumping between the ground state sublevels. Recipe: take into account excited state probabilities p_e in the large-detuning limit, as given by (2.4). The spontaneous emission rate is then γp_e. Spontaneous emission leads to ground state levels with relative probabilities ("branching ratio") given by the squares of the corresponding Clebsch-Gordan coefficients.[3] For example, for the state $|+1/2\rangle$, the probability is

$$p_{e,+3/2}(+) = \frac{D^2}{(\hbar\Delta)^2}|E_{+1}|^2$$

[3]Why: the standard calculation with Fermi's Golden Rule involves the squared electric dipole matrix element between the excited and ground state, and this is proportional to the CG coefficient.

in the excited state $|e, +3/2\rangle$ and

$$p_{e,-1/2}(+) = \frac{D^2}{3(\hbar\Delta)^2}|E_{-1}|^2$$

in the excited state $|e, -1/2\rangle$. Spontaneous emission from the state $|e, +3/2\rangle$ leads back to the same ground state, and we do not have to take it into account for a change in the ground state population. From the excited state $|e, -1/2\rangle$, the spontaneous emission of a linearly polarized photon leads to the other ground state $|-1/2\rangle$. This occurs in $\frac{2}{3} = \frac{2}{3}/(\frac{2}{3} + \frac{1}{3})$ of all cases. Hence the rate for optical pumping into this state is

$$\gamma_{+\to-} = \gamma\frac{2D^2}{9(\hbar\Delta)^2}|E_{-1}|^2$$

By a similar reasoning, we can find the rate for the reverse process:

$$\gamma_{-\to+} = \gamma\frac{2D^2}{9(\hbar\Delta)^2}|E_{+1}|^2$$

which is not the same since the amplitude of the electric field changes with position. One of the rate equation for the ground state populations is now

$$\dot{\rho}_{++} = -\gamma\frac{2D^2}{9(\hbar\Delta)^2}|E_{-1}|^2\rho_{++} + \gamma\frac{2D^2}{9(\hbar\Delta)^2}|E_{+1}|^2\rho_{--},$$

where the equation for ρ_{--} is redundant because the sum of both populations is conserved.

4.4.4 Steady state populations for an atom at rest

In our laser field, the pumping rates are given by

$$\begin{aligned}\gamma_{\pm\to\mp} &= \gamma\frac{2D^2}{9(\hbar\Delta)^2}|E_{\mp1}|^2 = \gamma'\left(1 \pm \sin 2kz\right) \\ \gamma' &= \gamma\frac{2D^2|\mathcal{E}|^2}{9(\hbar\Delta)^2}.\end{aligned} \tag{4.13}$$

Using $\rho_{--} = 1 - \rho_{++}$, the rate equation can be written in the form

$$\dot{\rho}_{++} = -2\gamma'\left[\rho_{++} - \frac{1}{2}\left(1 - \sin 2kz\right)\right] = -2\gamma'\left[\rho_{++} - \rho_{++}^{(ss)}\right]$$

with the steady state solutions

$$\begin{aligned}\rho_{++}^{(ss)} &= \frac{1-\sin 2kz}{2},\\ \rho_{--}^{(ss)} &= \frac{1+\sin 2kz}{2}.\end{aligned}$$

We note that these populations depend on the position of the atom. This is indicated in figure 4.3 by the large discs at the bottom of the potentials. At positions where the light is purely σ^--polarized ($\sin 2kz = 1$), the atoms are pumped into the state $|-\rangle$, and in steady state, 100% of the population accumulates there. Half a wavelength to the right, the atoms are pumped into the state $|+\rangle$. Note also that the steady state is reached on a timescale given by $1/2\gamma'$, which is smaller than the spontaneous lifetime $1/\gamma$. There is some "time lag" before the atom reaches equilibrium.

4.4.5 Average dipole force for atom at rest

The force for an atom at rest can be computed by taking the gradients of the light shift (interpreting these as mechanical potentials for the atom) and averaging with respect to the distribution on the sublevels. The result is

$$\begin{aligned}F(z,v=0) &= \left\langle -\frac{\mathrm{d}}{\mathrm{d}z}H_{\text{eff}}\right\rangle = -\sum_{\alpha=+,-}\rho_\alpha\frac{\mathrm{d}}{\mathrm{d}z}\langle\alpha|\,H_{\text{eff}}\,|\alpha\rangle\\ &= \frac{1-\sin 2kz}{2}\hbar k\Delta' 2\cos 2kz + \frac{1+\sin 2kz}{2}\hbar k\Delta'(-2)\cos 2kz\\ &= -2\hbar k\Delta'\sin 2kz\,\cos 2kz \qquad (4.14)\end{aligned}$$

The force vanishes for an atom at the bottom of either potential wells ($2kz = \pi/2,\, 3\pi/2$) and is a stable restoring force there if the detuning is negative (then also $\Delta' < 0$).[4] Hence, if we succeed to dissipate all kinetic energy, then the atoms gather in the potential wells with an alternating orientation of their spin states ("antiferromagnetic order"). But we have to ensure first that cooling is possible.

[4] The force also vanishes at $kz = 0$, for example, but this point is not stable as a positive displacement leads to a positive force, pulling the atom away from $z = 0$.

4.4.6 Atom in motion: friction force

For an atom in motion at the velocity v, we can change the optical pumping equation into the following form ($\alpha = +, -$)

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\rho_\alpha = -2\gamma'\left(\rho_\alpha - \rho_\alpha^{(ss)}\right)$$

where on the left hand side, a hydrodynamic derivative appears. The populations $\rho_\alpha(x,t)$ may now be interpreted as a "population field" depending on the instantaneous position in addition to time.

We are looking for the friction force to first order in v and in the stationary state in the laboratory frame. The explicit time derivative can then be dropped, and we find

$$\rho_\alpha = \rho_\alpha^{(ss)} - \frac{v}{2\gamma'}\frac{\partial\rho_\alpha}{\partial x}.$$

This is an implicit equation. To first order in v, we can replace on the right hand side the population ρ_α by its value for an atom at rest, i.e., by $\rho_\alpha^{(ss)}$. The resulting correction to the equilibrium population is

$$-\frac{v}{2\gamma'}\frac{\partial\rho_{\pm\pm}^{(ss)}}{\partial z} = \pm\frac{kv}{\gamma'}\cos 2kz$$

This correction gives rise to a friction force

$$F_{\text{Sis}}(z,v) = F(z,v=0) + \frac{kv}{\gamma'}2\hbar k\Delta'\,\cos^2 2kz$$

which is indeed opposite to the velocity if the detuning Δ is negative (then also $\Delta' < 0$). Typically, one takes the average over one wavelength of this force. The velocity-independent part (4.14) averages to zero and one finds an "effective friction coefficient" (positive if $\Delta < 0$)

$$\alpha = -\frac{\hbar k^2}{m}\frac{\Delta'}{\gamma'} = -\frac{\hbar k^2}{m}\frac{3\Delta}{\gamma}$$

Because of the factor Δ/γ, the friction can be much larger than the one for Doppler cooling (4.9) if the detuning is large.

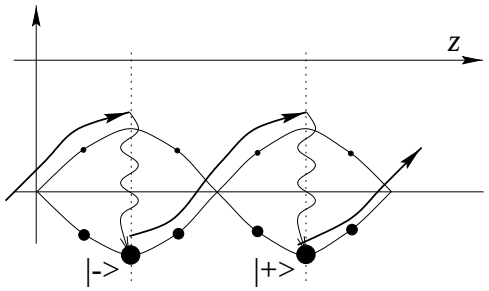

Figure 4.4: Mechanism of Sisyphus cooling. An atom in motion "climbs" its light shift potential until getting pumped into the other Zeeman sublevel.

4.4.7 Sisyphus

The interpretation of the strong friction in this configuration is as follows, see figure 4.4. Let us consider an atom that starts in the equilibrium state $|-\rangle$ at the bottom of its potential well and let us give this atom some velocity. Since optical pumping acts only with a certain time lag, the atom will stay in the state $|-\rangle$ and "climb" the corresponding light shift potential. It loses kinetic energy until, finally, an optical pumping process pumps it into the other state $|+\rangle$. In addition, this process occurs with the largest probability right at the maximum of the $\langle -| H_{\rm eff} |-\rangle$ potential where the loss of kinetic energy has been largest (see optical pumping rate $\gamma_{-\to+}(z)$, eq.(4.13)). The energy difference between the two light shift potentials is taken away by the spontaneously emitted photon: this is where the energy is dissipated into. And in the new state $|+\rangle$, which is now at the bottom of its potential, the cycle starts again if the atom still has some velocity. Repeating this over many periods, the atom always "climbs the hills" and loses its kinetic energy until it gets trapped in a well of the potential.