Chapter 2

Scattering forces

"Untersuchungen ber die Druckkräfte des Lichtes" by Peter Lebedew (Lebedew, 1901)

"Radiation in the solar system" by J. H. Poynting (Poynting, 1904a)

"Radiation in the solar system: its effect on temperature and its pressure on small bodies" by J. H. Poynting (Poynting, 1904b)

"Contributions of John Henry Poynting to the understanding of radiation pressure" by R. Loudon and C. Baxter (Loudon & Baxter, 2012)

"Radiation forces on small particles in the solar system" by Joseph A. Burns and Philippe L. Lamy and Steven Soter (Burns & al., 1979)

2.1 Photons and particle

Scattering event between 'photon' and 'particle' (or 'atom').

Energy conservation, elastic scattering: $|\mathbf{k}| = |\mathbf{k}'|$. (Exercise: recoil shift.)

Momentum conservation, momentum transfer:

$$\mathbf{p}'_A - \mathbf{p}_A = \hbar(\mathbf{k} - \mathbf{k}') \tag{2.1}$$

Average over many scattering events, collimated beam, isotropic re-emission

$$\overline{\mathbf{k} - \mathbf{k}'} = \mathbf{k} \tag{2.2}$$

Typical expression (Cohen-Tannoudji, laser cooling).

$$\text{radiation pressure force:} \qquad \mathbf{F}_{\text{rad}} = \hbar \mathbf{k} p_e \gamma_e \tag{2.3}$$

with scattering rate $p_e \gamma_e$. Background: two-level atom with states $|g\rangle$ and $|e\rangle$. Population/probability p_e of excited state. Decay rate γ_e, lifetime $\tau_e = 1/\gamma_e$ of excited state (like radioactive decay).

Order of magnitude: $\sim 10^5 \mathrm{m/s}^2$, huge compared to gravity. Typical number $\tau_e \sim 50\,\mathrm{ns}$, mass $M = 100\,\mathrm{amu}$, population $p_e \sim 1/2$, visible light wavelength $500\,\mathrm{nm}$.

Link to classical electrodynamics: scattering rate and (total) cross section

$$p_e \gamma_e = \frac{|\mathbf{S}(\mathbf{r})|}{\hbar\omega} \sigma_{\mathrm{tot}} \tag{2.4}$$

with Poynting vector $\mathbf{S}(\mathbf{r})$ (energy / area / time). Re-interpret as number of photons / area / time by dividing by $\hbar\omega$. Cross-section σ_{tot} is an effective area. Simple model of harmonically bound charge (electrodynamics lecture): $\sigma_{\mathrm{tot}} \sim \lambda^2$ possible on resonance.

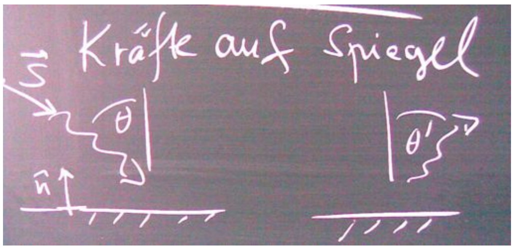
Re-write radiation pressure (2.3) with the Poynting vector for collimated beam ($\mathbf{k}$ and $\mathbf{S}$ are parallel):

$$\mathbf{F}_{\mathrm{rad}} = \mathbf{k} \frac{|\mathbf{S}(\mathbf{r})|}{\omega} \sigma_{\mathrm{tot}} = \frac{\mathbf{S}(\mathbf{r})}{c} \sigma_{\mathrm{tot}} \tag{2.5}$$

using the free space dispersion relation $\omega = c|\mathbf{k}|$. Hence interpret $\mathbf{S}(\mathbf{r})/c$ as a pressure (force/area) associated with the light beam.

Key concept from momentum balance Eq.(2.2): recoil momentum $\pm\hbar\mathbf{k}$ at absorption and emission events.

2.2 Photons reflected at a surface



Incident beam: angle θ with normal, reflected beam: angle θ' (need eventually an average for a diffusely reflecting surface).

Balance for normal component of momentum, absorption and re-emission

$$P_{\mathrm{abs}} = \frac{\hbar\omega}{c} \cos\theta, \qquad P_{\mathrm{em}} = \frac{\hbar\omega'}{c} \cos\theta' \tag{2.6}$$

Positive if a 'pressure force' (the light pushes the mirror).

Translated into pressure (need projected area, hence the $\cos\theta$) with incident Poynting vector $S = |\mathbf{S})$

$$p = \frac{S}{c} \cos\theta \left(\cos\theta + \cos\theta'\right) \tag{2.7}$$

Observe that both absorption and re-emission give a 'recoil push' in the same direction. This is different from the atom (particle) where the emission was isotropic.

Numerical value for sunlight, integrated over the entire spectrum. See problem sheet 1: the spectrum is in good approximation that of a black body. The resulting Poynting vector at the surface of Earth is known as the

$$\text{'solar constant'} \qquad S \approx 1367 \frac{\mathrm{W}}{\mathrm{m}^2} \tag{2.8}$$

so that the associated pressure (assuming that Earth is a specularly reflecting mirror, $\cos\theta' = \cos\theta$) is in order of magnitude (normal incidence, sun in zenith)

$$p \sim \frac{2S}{c} \sim 10^{-5}\,\mathrm{Pa} \tag{2.9}$$

This is negligible compared to the atmospheric pressure – and high up in the atmosphere where the pressure drops, there is no efficient reflection taking place.

You estimate in the problem sheet that this radiation pressure is also negligible when it is integrated over the cross section of the Earth – at least compared to the gravitational force between Sun and Earth. We shall see that radiation pressure becomes significant for smaller objects in the solar system.

Exercise: estimation for a small mirror and a cavity. Go back to photon picture, introduce 'round-trip time' $\tau = 2L/c$ with cavity length L. Assume that the mirror is moving in a harmonic potential with spring constant $M\Omega^2$ and estimate the displacement due to the radiation pressure for one photon, in units of the light wavelength.

Chapter 3

Continuous media

"The enigma of optical momentum in a medium" by Stephen M. Barnett and Rodney Loudon (Barnett & Loudon, 2010)

3.1 Radiation force on a liquid surface

Experiment performed in 1973 by Ashkin & Dziedzic (1973). Theory worked out by Lai & Young (1976). Our discussion follows closely Brevik (1979).

3.1.1 Experimental setting

For the typical parameters, see below in the text. A pulsed laser beam (wavelength $530\,\mathrm{nm}$), focused near the water surface, is incident from above. One observes that the water 'bulges' upward as the beam enters. This creates a convex lens that focuses the beam: one observes that the focal length is first very large, goes through a minimum $\mathcal{O}(200\,\mu\mathrm{m})$ and the focusing disappears again. The typical time scale is $\mathcal{O}(100 - 500\,\mathrm{ns})$, much longer than the duration of the laser pulse ($\sim 60\,\mathrm{ns}$).

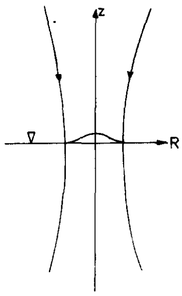


Fig. 6. Shape of beam focused on the free liquid surface.

3.1.2 Short times: sound waves

Hydrodynamic description for the first few ns of the pulse. Brevik focuses on the electrostriction force: this is a volume force with density

$$\mathbf{f}_{\text{es}} = \nabla\chi, \qquad \chi = \frac{1}{2}\mathbf{E}^2 \varrho \frac{\partial\varepsilon}{\partial\varrho} \tag{3.1}$$

where ϱ is the mass density and ε the dielectric function of the liquid. In the standard arguments for electrostriction, the derivative $\partial\varepsilon/\partial\varrho$ is taken at constant temperature. Due to the short time scales (a few ns), one may rather take the adiabatic regime (constant entropy), considering that the exchange of heat is too slow to enforce a constant temperature.

The force density on the liquid gives the equation of motion for the velocity field (the Euler equation)

$$\varrho \frac{\partial\mathbf{v}}{\partial t} = -\nabla p + \nabla\chi \tag{3.2}$$

where p is the hydrodynamic pressure. We neglect viscosity here. From Eq.(3.2), we see that the vorticity $\nabla \times \mathbf{v}$ can be taken as zero at all times. Hence, we can express the velocity in terms of a velocity potential Φ

$$\mathbf{v} = \nabla\Phi \tag{3.3}$$

and get from the Euler equation, up to a spatially constant reference value (for the pressure):

$$\varrho \frac{\partial\Phi}{\partial t} = -p + \chi \tag{3.4}$$

The equation of continuity is

$$0 = \frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho\mathbf{v}) = \frac{\mathrm{d}\varrho}{\mathrm{d}t} + \varrho\nabla\cdot\mathbf{v} \tag{3.5}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}\cdot\nabla$ is the co-moving (or substantial) derivative.

We take the substantial derivative of the Euler Eq.(3.4) and neglect terms quadratic in the velocity (sound waves of small amplitude). This gives on the left-hand side

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho\frac{\partial\Phi}{\partial t} \approx \varrho\frac{\partial^2\Phi}{\partial^2 t} \tag{3.6}$$

On the right-hand side, we express the pressure change in terms of the change in density, assuming that at each instant, we have an equation of state

$$-\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\mathrm{d}\chi}{\mathrm{d}t} = -\frac{\partial p}{\partial\varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}t} + \frac{\mathrm{d}\chi}{\mathrm{d}t} \approx +u^2\varrho\nabla\cdot\mathbf{v} + \frac{\partial\chi}{\partial t} \tag{3.7}$$

where in the last step, the continuity Eq.(3.5) was used. (We also assume that the speed of sound u, given by $u^2 = \partial p/\partial\varrho$ taken at constant entropy, is a constant.) The divergence $\nabla\cdot\mathbf{v}$ becomes a second derivative of the velocity potential so that we get:

$$\frac{\partial^2\Phi}{\partial^2 t} - u^2\nabla^2\Phi = \frac{1}{\varrho}\frac{\partial\chi}{\partial t} \tag{3.8}$$

This is a wave equation with a source term. The dispersion relation for the sound waves follows from a plane wave ansatz $\Phi(\mathbf{r},t) \sim \exp\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)$ as

$$\omega = u|\mathbf{k}| \tag{3.9}$$

The source term on the right-hand side (rhs) of Eq.(3.8) is the volume force density due to electrostriction. The boundary conditions at the water surface $z = h(x,y)$ is that the velocity there equals the change in height: $v_z(x,y,h) = \partial_t h(x,y)$. The simplest approximation to calculate the sound in the bulk of the water is to assume that the water surface remains flat $h = 0$. This gives $v_z(x,y,0) = \partial_z\Phi(x,y,0) = 0$, a so-called von Neumann boundary condition.

Brevik solves the inhomogeneous wave equation (3.8) with the 'method of images', using a gaussian beam approximation for the light intensity distribution in the liquid. One finds that the pressure at the surface, $p(x,y,0,t)$ in the center of the beam rises on a time scale $\mathcal{O}(w_0/u)$ given by the beam waist w_0 and the speed of sound. Typical numbers are $w_0/u \sim 2\,\mu\mathrm{m}/10^3\,\mathrm{m/s} = 2\,\mathrm{ns}$. Long after this time, sound waves leave into the bulk of the water, and the pressure equilibrates to compensate for the electrostrictive force density: $p = \chi$ in Eq.(3.4).

Longer times: surface force and tension

In the experiment of Ashkin & Dziedzic, the laser was pulsed with a time much longer than $1\,\mathrm{ms}$. What happens at longer times is that the water surface changes. To simplify the calculations, Brevik adopts the model of an ideal (no viscosity) and incompressible fluid. The density ϱ is then constant, and the pressure p becomes an independent dynamical variable. In addition, one has to apply boundary conditions for the force density at the surface that take into account surface tension. For completeness, we also include gravity here (natural if the pressure rises deep in the water).

For technical details on surface tension, see the lecture by John W. M. Bush from MIT (Boston), online at
http://web.mit.edu/1.63/www/Lec-notes/Surfacetension/Lecture2.pdf

The density of forces in the bulk of the water, incompressible and neglecting viscosity, becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}(\varrho\mathbf{v}) = \varrho\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = -\nabla p + \varrho\mathbf{g} \tag{3.10}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}\cdot\nabla$ is the convective derivative and $\mathbf{g} = -g\mathbf{e}_z$ the acceleration vector due to gravity. (We take the z-axis pointing upwards, from water to air.) When linearized for small $\mathbf{v}$, this equation gives $\mathrm{d}\mathbf{v}/\mathrm{d}t \approx \partial_t\mathbf{v}$.

We represent the velocity field again by a potential, $\mathbf{v} = \nabla\Phi$ and get from the equation of continuity

$$0 = \partial_t\varrho + \nabla\cdot(\varrho\mathbf{v}) = \frac{\mathrm{d}\varrho}{\mathrm{d}t} + \varrho(\nabla\cdot\mathbf{v}) = \varrho(\nabla\cdot\mathbf{v}) \tag{3.11}$$

so that we have the simple Laplace equation $\nabla^2\Phi = 0$.

When we integrate the force Eq.(3.10) along the z-axis, we get

$$\varrho\partial_t\Phi(\mathbf{r}) = -p(\mathbf{r}) - \varrho g z \tag{3.12}$$

This is value up to just below the surface at $z = h(\mathbf{x})$. We can use the freedom of choosing the integration constant to define that the pressure $p = 0$ at the surface.

In the water surface, we have to add the force densities due to the optical force and due to surface tension. The optical force per area is given by the difference in the electromagnetic stress tensors in air (dielectric constant 1) and in water ($\varepsilon = n^2$ with $n \approx 1.33$ in the visible range):

$$T_{zz} = \frac{\varepsilon_0}{2}(n^2 - 1)\mathbf{E}^2(\mathbf{x}, h, t) \tag{3.13}$$

where $\mathbf{E}^2(\mathbf{x}, h, t)$ is the light intensity evaluated at the surface. Note that this formula is the same for both viewpoints, by Minkowski and Abraham, on the energy-momentum tensor.

The surface tension σ depends on the curvature of the surface (see the lecture by Bush mentioned above):

$$\sigma = \gamma(\frac{1}{R_1} + \frac{1}{R_2}) = \gamma(\nabla_s \cdot \hat{\mathbf{n}}) \tag{3.14}$$

where γ is the surface tension and $R_{1,2}$ are the principal radii of curvature. The curvature is calculated from the unit vector $\hat{\mathbf{n}}$ normal to the surface (pointing from the liquid into air) and $\nabla_s \cdot$, the divergence evaluated along the surface. (This is natural since the vector field $\hat{\mathbf{n}}$ 'lives' only on the surface.) For clean water (no soap!), a typical value for the surface tension is $\gamma \approx 0.073\,\mathrm{J/m^2}$.

For a surface described by the height function $z = h(\mathbf{x})$, the normal vector can be constructed from the gradient

$$\hat{\mathbf{n}} = \frac{\nabla(z - h(\mathbf{x}))}{|\nabla(z - h(\mathbf{x}))|} = \frac{\mathbf{e}_z - \nabla_s h}{(1 + (\nabla_s h)^2)^{1/2}} \tag{3.15}$$

Expanding this to first order in h, we get for the curvature the expression

$$\nabla_s \cdot \hat{\mathbf{n}} \approx -\nabla_s^2 h \tag{3.16}$$

so that the surface tension is proportional to the second derivative of the height function.

Finally, by adding surface tension and the optical surface force to Eq.(3.12), the balance of surface stresses takes the form (recall that $p = 0$ at the surface)

$$\varrho \partial_t \Phi = -\varrho g h + \gamma \nabla_s^2 h + \frac{\varepsilon_0}{2}(n^2 - 1)\mathbf{E}^2(\mathbf{x}, h, t) \tag{3.17}$$

To complete the equations, we have to apply the boundary condition for the motion of the surface: the vertical velocity is equal to the rate of change of the height

$$\partial_t h(\mathbf{x}) = v_z(\mathbf{x}, h) = \partial_z \Phi(\mathbf{x}, h) \tag{3.18}$$

This is known as a 'free surface'. (A 'fixed surface' would correspond to $h = 0$, for example.)

Solving for the surface deformation

The hydrodynamic equations derived so far provide a description for water waves, also known as capillary waves. We shall make the small-amplitude approximation and continue to work to lowest order in h. This means, for example, that in the free surface condition (3.18) the velocity potential can be evaluated at $z = 0$: $\Phi(\mathbf{x}, h) \approx \Phi(\mathbf{x}, 0)$.

To analyze the water wave equation (3.17), we assume a solution in the form of a plane wave $\exp \mathrm{i}\mathbf{k} \cdot \mathbf{x}$ (here, $\mathbf{k}$ is a two-dimensional wave vector in the surface). From the Laplace equation in the bulk for Φ, we get

$$0 = \nabla^2 \Phi(\mathbf{k}, z) = -\mathbf{k}^2 \Phi + \partial_z^2 \Phi \tag{3.19}$$

This can be solved by exponentials $\mathrm{e}^{\pm kz}$ with $k = |\mathbf{k}|$. The physical solution that remains finite 'deep inside the water' is $\Phi(\mathbf{k}, z) = \Phi(\mathbf{k})\, \mathrm{e}^{kz}$. Evaluated at the (approximate) surface $z = 0$, this gives us the derivative $\partial_z \Phi = k\Phi$. The condition for the free surface therefore yields $\Phi = (1/k)\partial_t h$ so that the wave equation becomes

$$\varrho \partial_t^2 h + \varrho g k h + \gamma k^3 h = k I(\mathbf{k}, h, t) \approx k I(\mathbf{k}, 0, t) \tag{3.20}$$

where $I = \frac{1}{2}\varepsilon_0 (n^2 - 1)\mathbf{E}^2$ is the optical surface stress. We again used the small-amplitude approximation. This is a wave equation with a 'source term' provided by the optical stress. The left-hand side (lhs) of Eq.(3.20) gives the dispersion relation of the capillary waves: we assume a time dependence $h \sim \mathrm{e}^{-\mathrm{i}\omega t}$ and get

$$\omega^2 = gk + \frac{\gamma}{\varrho} k^3 = gk(1 + (k\delta)^2), \qquad \delta^2 = \frac{\gamma}{\varrho g} \tag{3.21}$$

The length δ is called the 'capillary length', its value for water and gravity at the surface of earth is $\delta \approx 2.7\,\mathrm{mm}$. For wavelength much larger than this, surface tension can be neglected, and we find the dispersion relation

$$k\delta \ll 1: \qquad \omega \approx \sqrt{gk}, \qquad v_g \approx \sqrt{\frac{g}{4k}} \tag{3.22}$$

Exercise. Get typical numbers for the group velocity v_g: waves on the beach, very-long wavelength waves on the ocean (note that the penetration depth into deep water is $\mathcal{O}(1/k)$).

The laser beam that excites surface waves in the experiment has a typical width w_0 of a few microns. In additional, the spatial profile is often a gaussian. Adopting the convention that w_0 gives the beam radius where the intensity has fallen by a factor $1/\mathrm{e}^2$, we get for the spatial Fourier transform the gaussian

$$I(\mathbf{k}, 0, t) = I(t)\, \mathrm{e}^{-k^2 w_0^2/8} \tag{3.23}$$

where $I(t)$ is proportional to the power of the laser beam and carries the time-dependence of the laser pulse. With this form, the wave equation to solve takes the form of a driven oscillator

$$\partial_t^2 h + \Omega_k^2 h = I(t) k\, \mathrm{e}^{-k^2 w_0^2/8} \tag{3.24}$$

where Ω_k is the capillary wave dispersion relation (3.21).

With a specific model for the pulse shape, Brevik works out the solution to Eq.(3.24). The observable that he finally calculates is related to the curvature of the water bulge: indeed, as the water is pulled upwards, its curved surface acts like a collecting lens that focuses the laser beam. Adopting the approximation of a spherical lens, geometrical optics yields the inverse focal length

$$\frac{1}{f} = \frac{n-1}{n}\frac{1}{R} \approx -\frac{n-1}{n}\nabla_s^2 h \tag{3.25}$$

where R is the radius of curvature and n the index of water relative to air. From the Fourier expansion of h, we get on the beam axis

$$-\nabla_s^2 h(0,t) = \int \mathrm{d}^2k\, k^2 h(\mathbf{k},t) \tag{3.26}$$

The result is shown here:

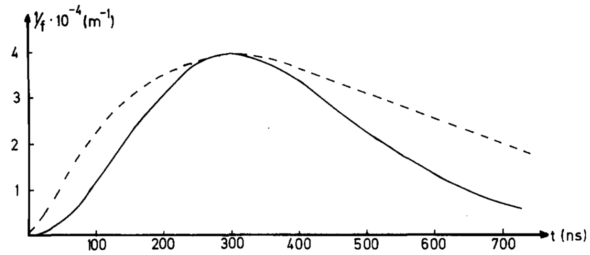


Fig. 7. Time development of the inverse focal length $1/f$. Full line gives the theoretical result calculated from (4.43) with the effective beam radius $w_0 = 4.5\,\mu$m. Broken line gives the experimental result of Ashkin and Dziedzic [8] displaced by 60 ns to the left.

The typical focal length (maximum of the curve) is of the order of $(1/4)10^{-4}\,\mathrm{m} = 25\,\mu\mathrm{m}$. It is reached after a time $\mathcal{O}(300\,\mathrm{ns})$. If we estimate the second derivative in Eq.(3.25) by a factor $1/w_0^2$, we get an elevation of the surface of the order of $\max h \sim w_0^2/f \sim 0.1 - 1\,\mu\mathrm{m}$, depending on the waist w_0 and the numerical factor involving the index n. This is relatively small and difficult to detect directly. But it roughly validates our assumption that the height h is 'small' compared to the spot size w_0. The period of capillary waves with $k = 2/w_0$ is of the order of $150\,\mathrm{ns}$ which is roughly the time scale for the build-up in the figure above.

These numbers correspond to an experiment performed with a beam of power $3\,\mathrm{kW}$ (quite high) and waist $w_0 = 2.1\,\mu\mathrm{m}$. For a quantitative evaluation, one has to take into account that a part is reflected from the water surface: one needs the field intensity right at the interface which gives another factor depending on the index n.

Brevik compares in detail theory and experiment: the maximum of the inverse focal length is displaced in time with respect to the experimental work – this may be related to the 'timing' of the measurement and the uncertainty in determining the 'start' of the pulse. The value for $1/f$ comes out a factor

$\mathcal{O}(10)$ too large – this can be improved if a larger value for the beam waist $w_0 \approx 4.5\,\mu\text{m}$ is taken. And of course, the profile of the bulge is not exactly gaussian so that the approximation of a spherical lens is questionable. In particular, small-scale water waves may lead to scattering and reduce the light intensity. This may explain the observed halo of forward scattered light. These qualitative and quantitative considerations are enough to be happy with the comparison to experiment.

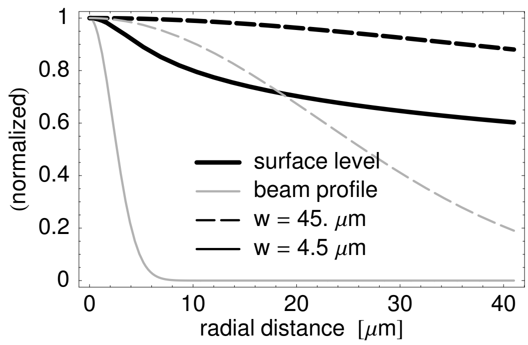


Figure 3.1: Spatial profile of the 'bulge' pulled out of a water surface by a laser with waist w_0, under stationary conditions. Black: water profile; gray: (gaussian) laser intensity. Solid lines: waist $w_0 = 2.1\,\mu\text{m}$, dashed lines: larger waist $w_0 = 4.5\,\mu\text{m}$. The capillary length is $\delta = 2.73\,\text{mm}$.

Let us finally consider the case that the light beam is not pulsed, but has a continuous intensity I_0 (at the surface). Ignoring the absorption of light, one can focus on the stationary situation where the height does not change any more. We then get the Fourier transform

$$h(\mathbf{k}) = \frac{I_0 k\, \mathrm{e}^{-k^2 w_0^2/8}}{\Omega_k^2} = \frac{I_0}{\varrho g} \frac{\mathrm{e}^{-k^2 w_0^2/8}}{1 + k^2 \delta^2} \tag{3.27}$$

Transformed into real space, this gives (in polar coordinates, the angular integral can be performed, giving the Bessel function $J_0(kx)$)

$$h(\mathbf{x}) = \frac{I_0}{2\pi \varrho g} \int_0^\infty \mathrm{d}k\, J_0(kx) \frac{k\, \mathrm{e}^{-k^2 w_0^2/8}}{1 + k^2 \delta^2} \tag{3.28}$$

Examples of this spatial profile are plotted in Fig.3.1 below for different choices of the laser waist w_0 relative to the capillary length. The profiles are normalized to unity at the beam center. Note that they differ quite significantly from the sketch in Brevik's Fig.6 above: the water profile is much wider than the width of the laser beam. It is as if the laser were pulling on a tissue: due to the surface tension, the fluid follows the central pull over a wide radial range.

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