Einführung in die Quantenoptik I

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Hinweis. Die Übungsaufgaben sind ein Versuch, verschiedene 'Geschmäcker' zu bedienen: mal geht es um Abschätzungen, Einheiten, Größenordnungen. Mal gibt es einiges zu rechnen. Häufig ist schon das Interpretieren des Aufgabentextes Teil der Herausforderung. Und dann gibt es noch *soft skills* wie das Erstellen von Texten, Recherche von Literatur oder anderer Information im Netz.

Es gilt die Regel: Lassen Sie sich von Fehlern in den angegebenen Formeln nicht verwirren. Im Zweifelsfall fehlt eben im Aufgabentext ein Faktor 2, π, i, -1 ...

Problem 1.1 – Typical numbers (7 points)

(i) Calculate the frequency $f = \omega/2\pi$ and the photon energy $hf = \hbar\omega$ for visible light of your favorite colour. Give the frequency (energy) in THz, eV, cm^{-1} ('wave numbers'), kcal/mol (for chemists only), and in units of $k_B T$ with room temperature T. Look up the power of a typical hand-held laser pointer and translate it into a photon flux.

(ii) Calculate the ratio between the size of your favorite atom and the light wavelength from (i). What is smaller? Formulate in words what this means when this atom is placed in an electromagnetic field of this colour.

(iii) In the lecture, we are going to see that the interaction of an atom with light depends on the electric field $\mathbf{E}(\mathbf{r})$ at the position $\mathbf{r}$ of the atom. Estimate the magnitude of the electric field of a laser pointer and of the high-power laser of the *National Ignition Facility* in the U.S.A. (Ask around or think yourself about the link between the electric field and other typical parameters: light power, beam diameter etc.) Estimate as well the electric field created by the nucleus somewhere 'inside' your favorite atom and compare. Which field is smaller? What do you expect for the ratio between the interaction energy of the atom with the laser pointer and the typical energy levels of the atom?

Problem 1.2 – Quantum language (8 points)

In the quantum mechanics lecture, you have learned about the stationary states of a harmonic oscillator. These play a key role for quantum optics.

(i) A typical quantum state of a harmonic oscillator can be written in the form

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle = (c_0, c_1, \ldots)^T \tag{1.1}$$

Explain the meaning of the symbols in the three expressions. Express the Dirac kets $|0\rangle$, $|5\rangle$ as column vectors. Write down a formula for the wave function of the oscillator, assuming that the states $|n\rangle$ correspond to stationary states $\phi_n(x)$. What are the "normalization conditions" for these states (and wave functions)?

(ii) Explain the following statement: "For a quantum system in a finite-dimensional Hilbert space, physical observables are represented by hermitean matrices."

(iii) Consider the following observable

$$N = \begin{pmatrix} 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \sum_{n=0}^{\infty} n|n\rangle\langle n| \tag{1.2}$$

and check that the two expressions (matrix and ket-bra notation) are consistent by working out the expectation value $\langle N \rangle = \langle \psi | N | \psi \rangle$ in the state given in Eq.(1.1). Why does the name "excitation number" match the physical interpretation of this observable?

(iv) Take a state with nonzero c_0, c_1 and repeat the previous exercise for the oscillator displacement

$$X = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots \\ 1 & 0 & \sqrt{2} & 0 & \dots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \dots \\ 0 & 0 & \sqrt{3} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = a + a^\dagger \tag{1.3}$$

where a and $a^\dagger$ are the ladder operators. Questions for the physical interpretation: why is a alone not a real "observable"?

Problem 1.3 – Unitary beam splitter (5 points)

In the lecture, we have seen that a beam splitter mixes two beams according to the amplitude laws

$$a \mapsto A = ta + rb\,, \qquad b \mapsto B = t'b + r'a \tag{1.4}$$

where a, b, A, B are the photon annihilation operators. Make a sketch and write down the interpretation of the beam splitter amplitudes r, t, t', r'. Find out what conditions must be imposed on these amplitudes such that the commutators $[A, A^\dagger]$, $[A, B]$, $[A, B^\dagger]$ ... have the same structure as for the 'incident' photon operators a, b.