Einführung in die Quantenoptik I

Wintersemester 2015/16
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Übungsaufgaben Blatt 7

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Problem 7.1 – Scully-Lamb laser theory (8 points)

Marlan O. Scully and Willis Lamb have proposed in the 1960s a simple rate equation model for a laser. The probability p_n of finding n photons in the laser mode is assumed to evolve under gain by the active medium as follows:

- with a rate $G(n)(n+1)$, one photon is added = the laser jumps to the state with $n+1$ photons
- with a rate $G(n-1)n$, one photon is added to the state with $n-1$ photons, and the laser jumps to $|n\rangle$

Here the n-dependent function $G(n)$ describes the gain of the active medium, including the effect of saturation.

(1) Formulate similar stories for the process of 'a photon leaves the laser cavity' and show that both processes together yield the rate equations

$$\frac{\mathrm{d}p_n}{\mathrm{d}t} = -G(n)(n+1)p_n - \kappa n p_n + G(n-1)np_{n-1} + \kappa(n+1)p_{n+1} \tag{7.1}$$

Complete the equations for the lowest states $n = 0, 1$.

(2) In the stationary state, one can find the photon distribution p_n from the principle of detailed balance: the rates for jumps up and down between the states $|n\rangle$ and $|n+1\rangle$ are exactly equal. Show that this leads to

$$G(n)(n+1)p_n = \kappa(n+1)p_{n+1} \tag{7.2}$$

(3) Using the simple saturation model $G(n) = G_0/(1+Bn)$, show that the photon statistics is given by (no guarantee that this formula is exact)

$$p_n = p_0 \frac{G_0^n}{\kappa^n} \prod_{k=1}^{n} \frac{1}{(1+Bk)} \tag{7.3}$$

and make a sketch for $G_0 > \kappa$. Why is this called 'above threshold'?

Problem 7.2 – Lindblad master equation and Heisenberg picture (6 points)

The equation of motion for the density operator ρ of a laser cavity with a loss rate κ can be written in the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\left[H, \rho\right] + \kappa a \rho a^{\dagger} - \frac{\kappa}{2}\left\{\rho a^{\dagger} a + a^{\dagger} a \rho\right\} \tag{7.4}$$

where H is the cavity Hamiltonian. This is called a 'master equation in Lindblad form'.

(1) Show that the master equation preserves the trace of ρ.

(2) Show that the average value $\langle a \rangle$ of the field operator and the average photon number $\langle n \rangle = \langle a^{\dagger} a \rangle$ evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle a \rangle = -(\mathrm{i}\omega_c + \frac{\kappa}{2})\langle a \rangle \tag{7.5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n \rangle = -\kappa \langle n \rangle \tag{7.6}$$

provided the cavity Hamiltonian H has a suitable form (which one?).

Problem 7.3 – Casimir pressure and nanotechnology myths (6 points)

(1) Find out on the web what is the Casimir effect and how it can be understood in a simple ('heuristic') way.

(2) Check out the keyword 'stiction' and write a few sentences about the role of Casimir and van der Waals forces for nano-machines.

(3) Find out under which conditions it is possible to have Casimir repulsion. Who is citing Timothy H. Boyer [*Phys. Rev. A* **9** (1974) 2078] and what result did he get?