van der Waals- und Casimir-Kräfte

Wintersemester 2015/16

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Übungsblatt 1

Ausgabe: Fr 23 Okt 15
Eingabe: Do 29 Okt 15

Problem 1.1 – Defying gravity: geckos, insects, dust (7 Points)

The gecko that clings to a wall is a very popular example of van der Waals forces, but smaller animals like insects probably use similar techniques. (1) Using the typical numbers for the van der Waals interaction potential (look up 'Lennard-Jones potential'), estimate the force F for a pair of atoms or molecules in a distance range of $1 \ldots 10\,\mathrm{nm}$. How many pairs of molecules are needed for a gecko (an insect) to compensate for its weight?

(2) We shall see in the lecture that the thin hairs of a gecko (or an insect) are actually mesoscopic bodies and that van der Waals forces between them are proportional to their 'contact area'. Look up the wikipedia site on van der Waals forces and try to make an estimate how many hairs (per area) and of which size are needed to suspend the gecko (or an insect).

Problem 1.2 – Van der Waals equation of state (8 Points)

(1) Check the relation given in the lecture between the van der Waals parameters a, b and the second virial coefficient B_2:

$$B_2 = b - \frac{a}{kT} \tag{1.1}$$

(2) Analyze a van der Waals gas with a hypothetical repulsive interaction $W(\mathbf{x}, \mathbf{x}') = +c_6/|\mathbf{x} - \mathbf{x}'|^6$. Plot the 'Meyer function'

$$f(r) = 1 - \mathrm{e}^{-W(r)/kT} \tag{1.2}$$

and show by a suitable substitution that the virial coefficient goes like

$$B_2 = (\cdots)\sqrt{\frac{c_6}{kT}} \tag{1.3}$$

where $(\cdots)$ is a numerical coefficient.

(3) On the web, there are many details to find on the more conventional case of an attractive van der Waals interaction with a hard core: summarize the results for the virial coefficient B_2 and the parameters a, b in the van der Waals gas theory.

Problem 1.3 – Working with Boltzmann (5 Points)

A system in thermal equilibrium can be described by the so-called Boltzmann factor $\exp(-V/kT)$ where V gives the energy of the relevant state. A well-known example is the 'barometric formula' that gives the density of particles in the atmosphere vs. height. Let us play with the Boltzmann factor for a classical model of a molecule with a permanent dipole moment.

(1) The permanent dipole is given by a vector $\mathbf{d} = d\,\hat{\mathbf{n}}$ whose length d is fixed, while $\hat{\mathbf{n}}$ is a unit vector. A parametrization in spherical coordinates may be useful. In the simplest case, all directions appear with the same probability – this corresponds to $V = \text{const.}$ in the Boltzmann factor. Calculate the average values

$$\langle \mathbf{d} \rangle = 0 \tag{1.4}$$

$$\langle \mathbf{d}^2 \rangle = d^2 \,, \qquad \langle d_x d_z \rangle = 0 \,, \qquad \langle d_z^2 \rangle = \frac{d^2}{3} \tag{1.5}$$

(2) In a (static) electric field, one has to include the dipole interaction $V = -\mathbf{d} \cdot \mathbf{E}$ in the Boltzmann factor. Argue that the average dipole is now nonzero and parallel to $\mathbf{E}$. Write down an integral that gives $\langle d_z \rangle$ when the z-axis is parallel to $\mathbf{E}$. [Bonus points: compute the integral numerically for different values of the dimensionless parameter dE/kT.]

(3) Look up the keyword 'orientation polarization' in Jackson's book and check that for weak electric fields, the response of the average dipole is linear in the field:

$$\langle \mathbf{d} \rangle = \alpha \, \mathbf{E} \tag{1.6}$$

The coefficient α is called the polarizability.