

**Problem 5.1** – Atomic beam experiments (10 Points)

In the lecture, we have seen a few examples of atomic beam experiments that are sensitive to van der Waals forces between atoms and a surface. Let us try to figure out some numbers.

(1) The diffraction of Helium atoms can be used to detect the atomic structure of crystalline and amorphous surfaces. Find out the kinetic energy of a helium atom such that its de Broglie wavelength is equal to the typical unit cell of a crystal, 1 Å, say. Converted into temperature units, these beams are called ‘thermal energy helium atom beams’ [Ellis and the Toennies group, *Phys Rev Lett* **75** (1995) 886].

(2) At which angle of incidence do you get a ‘projected kinetic energy’ comparable to the binding (physisorption) energy ( $\sim 10$  meV) in the van der Waals potential? Keyword ‘Beeby correction’ [*J Phys C* **4** (1971) L359].

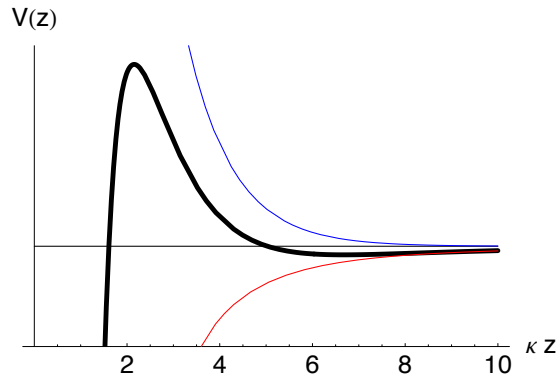
(3) The experiment of Sandoghdar and the Haroche group [*Phys Rev Lett* **68** (1992) 3432] is based on the angular deflection of an atom beam that ‘grazes’ a surface. The deflection can be calculated approximately by integrating the force over the typical flight time near the surface. Take a typical atom–surface distance of  $z = 10$  nm, a flight length of  $x = 1$  mm and the atomic beam data of item (1). Use a typical value for the der Waals potential  $-c_3/z^3$  (see also item (4) below). Which angular deflection do you expect? What kind of ‘beam collimation’ is needed to resolve this?

**Problem 5.2** – Ultracold atom ‘trampoline’ (10 Points)

The ‘ultracold atoms’ of the experiment by Landragin and the Aspect group [*Phys Rev Lett* **77** (1996) 1464] is based on atoms that fall down under gravity and are reflected at a surface. The surface must be ‘coated’ with a reflective potential whose height is at least this value. Take the simple model

$$V(z) = V_0 e^{-\kappa z} - \frac{\hbar\gamma}{(k_A z)^3} \quad (5.1)$$

where  $\hbar\gamma/k_A^3$  is a parametrization of the van der Waals atom+surface interaction constant  $c_3$ :  $\gamma$  is comparable to the linewidth of the D1 line (589 nm) of sodium, and  $k_A = 2\pi/\lambda$  its wavenumber. (1) Find a strategy (possibly numerical) to determine the position and height of the potential barrier (i.e., the maximum) and of the minimum.



**Figure 5.1:** Sketch of the ‘trampoline potential’ of a light-coated surface. Thin red: van der Waals potential; thin blue: repulsive ‘coating’.

(2) To find typical numbers for item (1), here are some formulas that my colleagues have used. The height of the barrier ( $V_0$ ) is a few times larger than the kinetic energy  $E_z$  an atom like sodium acquires when it falls over 1 cm. The  $c_3$  coefficient of the van der Waals potential can be parametrized as (compare the numbers from the two expressions, a convenient unit is meV nm<sup>3</sup>)

$$c_3 \simeq \frac{\hbar\omega_A\alpha_0}{32\pi\epsilon_0} \frac{n^2 - 1}{n^2 + 1}, \quad c_3 \simeq \frac{\hbar\gamma}{k_A^3} \quad (5.2)$$

Here, you make take  $\omega_A = k_A c$  as the Bohr frequency of the D1 and D2 lines,  $\alpha_0$  is the static polarizability and  $\gamma$  is comparable to the linewidth of the D1 or D2 lines. ([www.steck.us/alkalidata](http://www.steck.us/alkalidata) is an exhaustive data source for alkali atoms like sodium.)

(3) The surface parameters enter via the refractive index  $n$  of the surface material in the frequency region around the D1 line, and the decay constant  $\kappa$  of the ‘coating’. In the experiment, one uses the total internal reflection of a laser beam from the ‘inner side’ of the surface. The laser frequency is tuned near the atomic resonance. If the angle of incidence  $\theta$  is larger than a critical angle, show that one has  $\kappa = 2k_A(n^2 \sin^2 \theta - 1)^{1/2}$ .