Einführung in die Quantenoptik I

Wintersemester 2016/17
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Problem Set #1 Hand in: 25 October 2016
Discussion: 27 October 2016

Note. The problems try to address different preferences: sometimes they are about estimates, units, orders of magnitude; in others you have to calculate a few things. Quite often a part of the challenge is to understand the questions. Finally, you will be trained in a few soft skills like drafting texts, searching the literature or other information on the Web.

First rule: Don't get irritated by errors in the formulas. In case of doubt ('my result looks complicated'), there is an error in the problem sheet like a missing factor -1, 2, π, i . . .

Problem 1.1 – Typical numbers (8 points)

(i) Translate the power of a laser pointer into its electric field. Find out typical numbers for a laser pointer and for high-power lasers like those of the *National Ignition Facility* (USA). [Bonus question: Why does the *NIF* work with huge beam diameters?]

(ii) Argue that for a 'typical molecule' (not too large), the electric dipole moment is of the order of $d \sim ea_0$ where e is the electron charge and a_0 the Bohr radius.

'Argue' means: write down hand-waving arguments.

(iii) The interaction energy of an electric dipole $\mathbf{d}$ in a field $\mathbf{E}$ is given by $V_{\rm int} = -\mathbf{d} \cdot \mathbf{E}$. Estimate the laser power where this energy is comparable to the photon energy $\hbar\omega$, for a 'typical' atom or molecule.

(iv) Experiments in ultrafast optics here in Potsdam work with light pulses whose energy (per pulse) is in the $1\,\mathrm{mJ}$ range, with a spectrum that peaks in the visible.

Check out details with the groups of M. Bargheer and M. Gühr: find where their rooms are and ask somebody from the group, or check out their web pages.

How many photons does such a pulse contain? Imagine that the pulse irradiates a metallic surface and is absorbed there: how deep is the penetration depth of the light in the metal? Imagine that the light pulse is tightly focused and gets absorbed in a volume of $(100\,\mathrm{nm})^3$. If the atoms in this volume redistribute the absorbed energy among them (typically by electron-electron collisions), how big is the increase in energy per atom? What increase of temperature does this represent?

Problem 1.2 – Atoms and molecules are 'small' (8 points)

(i) Calculate the ratio between the size of your favourite atom and the wavelength of your favourite colour. What is smaller? Formulate in words what this means when this atom is placed in an electromagnetic field of this colour.

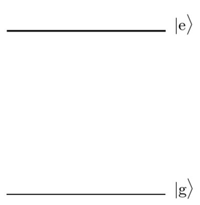
(ii) Look up the formulas from QM I that give the 'typical size' a of the hydrogen atom and the wavelength λ of its 'typical' optical transitions. Show that the ratio a/λ is of the order of the Sommerfeld fine structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$.

(iii) The magnetic dipole moments μ of 'typical' atoms and molecules are given by the product of an angular momentum or spin and the 'Bohr magneton'. The magnetic interaction energy in a field $\mathbf{B}$ is given by $V_{\rm m} = -\boldsymbol{\mu} \cdot \mathbf{B}$. Look up the definition of the Bohr magneton and show that in a laser beam, the ratio between magnetic and electric interaction energies is of the order of the fine structure constant as well:

$$\frac{V_{\rm m}}{V_{\rm e}} \sim \alpha \tag{1.1}$$

Use Maxwell equations to relate electric and magnetic fields of the laser. A plane wave is a useful approximation.

Problem 1.3 – Quantum notation (4 points)



The sketch on the left is often used for two-level systems, or 'q-bits' (qubits, qbits) in modern jargon. They can be implemented with different physical systems, leading to a variety of notations.

(i) If α, β are the probability amplitudes of the two states, write in Dirac bra-ket notation a general superposition state.

(ii) If one deals with a spin-$1/2$ system, the q-bit states are often called 'spin up' $|{\uparrow}\rangle$ and 'spin down' $|{\downarrow}\rangle$. Can you find out which amplitudes α, β are needed to make a superposition which is a 'spin left' $|{\leftarrow}\rangle$ state?

(iii) If the levels are electronic states of an effective 'one-electron system' like an alkali atom, write down the electronic wave function ψ for a generic superposition state.