Einführung in die Quantenoptik I

Wintersemester 2017/18
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Übungsaufgaben Blatt 5 Ausgabe: 09 Januar 2018
Eingabe: 18 Januar 2018

Aufgabe 5.1 – Thermal radiation, qualitatively (12 Punkte + 5 bonus Punkte)

For the following discussion, you may use that a temperature T corresponds to a characteristic wavelength $\lambda_T = \hbar c / k_B T$ (Wien's displacement law), at least for massless particles. In thermal equilibrium, these particles appear with an energy density $u \sim k_B T / \lambda_T^3$.

(1) Compute the average photon number at room temperature for visible light and for the frequency band used by your mobile phone.

(2) When the Universe was about three minutes old, it was in thermal equilibrium so that the thermal energy $k_B T_e$ was comparable to the rest mass of the electron. Estimate this temperature and the corresponding energy density. Which were the most abundant particles in this epoch?

(3) Consider protons and neutrons at the temperature of (2) and speculate why the ratio $\exp[(m_n - m_p)c^2/k_B T_e]$ gives an idea of the ratio between proton and neutron densities. How big is the number?

(4) Find out precise values of the photosphere temperature of the Sun and calculate the corresponding thermal wavelength $\lambda_\odot$. In which frequency band does this fall?

(5) Convince yourself that the energy $\Delta\dot{q}$ the Earth receives from the Sun by radiation (per day, per area) is exactly balanced by the (thermal) radiation of the Earth. What is not exactly balanced is the entropy $\Delta\dot{s} = \Delta\dot{q}/T$ (Clausius, written as a current density). Look up the keyword 'solar constant' and estimate the amount of entropy the Earth is losing into space (per day, per area). Normalized to $k_B \log 2$, this is the amount of information the Earth is receiving from the Sun (per day, per area). Compare with the amount of information in the genetic code of the biomass on Earth and estimate a time scale for the 'creation' of this information due to Sunlight. (Attention, this is a controversial subject.)

Boltzmann had already noted this in 1875: "The general struggle for existence of animate beings is not a struggle for raw materials—these, for organisms, are air, water and soil, all abundantly available—nor for energy which exists in plenty in any body in the form of heat, but a struggle for [negative] entropy, which becomes available through the transition of energy from the hot sun to the cold earth" [in "The Second Law of Thermodynamics", quoted by H. M. Nussenzveig, *Phys. Scr.* **91** (2015) 118001].

(6) [5 bonus] Electrons, protons, and neutrinos are fermions. Each mode (momentum state incl. spin label) can only be occupied by zero or one particle. Think about the partition function for massless neutrinos and argue that their density of free energy is given by

$$\frac{F_\nu}{V} = -k_B T \frac{2 \times 4\pi}{(2\pi\hbar)^3} \int_0^\infty \mathrm{d}p\, p^2 \log(1 + \mathrm{e}^{-cp/k_B T}) \tag{5.1}$$

Make an estimate for the energy density of a "cosmological neutrino background" at $T_\nu \approx 1\,\mathrm{K}$.

Aufgabe 5.2 – Vacuum energy (8 Punkte)

In the energy of the quantized radiation field, we often 'neglect' the contribution of the zero-point energy of each field mode, given by $\frac{1}{2}\hbar\omega_k$ per mode. This 'small term' leads, in the vacuum state of the electromagnetic field, to an energy density $u_{\rm vac}$ that depends on the cutoff wavenumber k_c as follows

$$u_{\rm vac} = C\hbar c k_c^4 \tag{5.2}$$

where C is a numerical constant 'of order unity'.

(0) Check that Eq.(5.2) is consistent with the rule given in Problem 5.1 provided a "vacuum temperature" $T_{\rm vac} = \hbar c k_c$ is introduced.

(1) Fix the cutoff at the energy scale $E_{\rm GUT} = \hbar c k_c$ for the 'unification' of the fundamental interactions (electroweak and strong, except gravity, look up the keyword 'grand unified theory') and make an estimate for the corresponding vacuum energy density. Express this number in atomic mass units (times c^2) per cubic meter. [Hoax: "this solves the energy crisis."]

(2) Look up the length scale $\ell_{\rm Planck}$ for the unification of gravity, relativity, and quantum theory and compare the vacuum energy corresponding to $\ell_{\rm Planck} = 1/k_c$ to the observed mass density in the Universe and the density of 'dark energy' (responsible for the accelerated expansion of the Universe, Nobel prize in physics 2011). [Jargon: "wrongest formula in all physics."]

For an informal exposition of this so-called "cosmological constant problem", see Adler, Casey, and Jacob [*Am. J. Phys.* **63** (1995) 620]. For a recent attempt to solve it, see Wang, Zhu, and Unruh [*Phys. Rev. D* **95** (2017) 103504 = arxiv:1703.00543].