Quantum Information (WiSe 2019/2020)

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Problem Set No 1 ($20+\pi$ scores)[1]

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▷ Aufgabe 1 (Queen on board) (2 scores)

To localize a single Queen on a 8×8 board of chess you need Y/N replies to

- ☐ $64/2 = 32$ questions on average
- ☐ $\sqrt{64} = 8$ questions
- ☐ exactly $\log_2(64) = 6$ questions

Remark: This is a classic which you are hopefully familiar with from your school-days ...

▷ Aufgabe 2 (Two fair dice) (2 scores)

In rolling a fair dice twice, it is

- ☐ more likely
- ☐ equally likely
- ☐ less likely

that the sum of the face values is as prime number than it being divisible by three.

▷ Aufgabe 3 (Run-Test) (6 scores)

In a N-digit binary string, a (maximal) run is a sequence of adjacent equal digits. For example, the 20-digit binary string `10011111010100001101` comes with 11 runs: The first run `1`, then the run `00`, then the run `11111`, and so on.

(a) How many N-digit binary strings come with exactly r runs?

(b) With the digits being produced in equal probability by a memoryless source, what ist the expectation value for the number of runs for N-digit binary strings?

Remark: The "Run-Test" is sometimes used to check whether a long string could have been produced by a "source with mermory" rather than by a memoryless source.

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[1]Problems with transcendental scores are facultative nuts. Nuts have high nutrition value ...

▷ Aufgabe 4 (Is this sequence random?) (10 scores)

Consider the list of bits given below. Could this be a sample from a random 0/1-source? Is there a reason to doubt that?

```
1100010000000101101010101111000110110011011110100110110001100010111111000000111101011011111111001010111000011111001100000100101
0110111110010101011100010100001010101001010101010111010010100000011101101110101100011101111011000111100000000110011010000011001
1010111101101001110000001101111110010111111001110101001000001010000010111101010011101000100111001100000100000000110111001 1001
0001011010000111001110110000000101110001001011101101010011010000000011011100000010101110011111101001011110000111111100100110101
0111111111000010011111001010111100001100010001111000100011010011100001011001011000110000111001010010010001110100011001011 0001
1001010101100010001010111110111100010100010001110110100010011111010101001100110100010001011010111011010110010001000101000 1011
1011101100001001111001001100100110000011100110101100001100000011010101111110111111111110101100000111000101010100011010100 0111
1011000011000011010010111101011010011100111010000011011001110011011010001111100011000110100101011101001010000011110111111 0011
1010010010101101001111001101101011000000101000011111010101000100110100111000000000000011001010011110101111000000011000110 01000
0111010000111111101000000101011000110011100110000100100110111110110111100010110001100100000000000000101011000011110100010000000
0111010011010100001111000011011110101110100100011110111000101011010011100100001100000101010100101000001011000010000000001 10000
0001001000100100110000110001000001001100010100110110000100010001010100000101101110011011001011010011011100000111010101100 100001
1101000100100000101001000101010101010001110101110100110001110000100100011110000000101011100001000110001100011000101110000 10110
1010010101001110001111001000111100000000000101100011100001101110100011001010101100100101001101011001011111001000111011100 10000
0001000111010000011111010000100011010100010100100110011011011011110100000111110100000001000001000100101001010001010011011 01010
1100000000100010000000110001011001111101000010001000101001010001110011001000110101001001111000011011010001111010000001001 010100
```

▷ Aufgabe 5 (Benford's Law) (π scores)

Look at the numbers in any newspaper, more specifically at all the numbers anywhere in the text, which are larger than 1000, and are *not* years, prices of goods in advertisements, or telephone numbers (but, for example, statistical data, like number of citizen in town etc). Take a note of the first digit of each number, and count the relative frequencies of the digits.

"Benford's Law" claims that the digit "1" appears in such an ensemble with frequency $\approx 30\%$, whereas "9" appears only with frequency $\approx 5\%$. Confirm and explain.

Hint: If you have no newspaper at hand, make the following experiment: think of a large number N (like $N = 277465890$); write down all numbers between 0 and N; determine the relative frequency of those numbers, whose leading digit is 1; repeat the experiment a couple of times (say 100 times) in order to accumulate some statistics. This should confirm the Benford law. But still – you have to think of an explanation ...