Einführung in die Quantenoptik I

Wintersemester 2019/20
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Problem Set #1 Hand Out: 15 October 2019
Hand In: 22 October 2019

Note. The problems try to address different preferences: sometimes they are about estimates, units, orders of magnitude; in others you have to calculate a few things. Quite often a part of the challenge is to understand the questions. Finally, you will be trained in a few soft skills like drafting texts, searching the literature or other information on the Web.

First rule: Don't get irritated by errors in the formulas. In case of doubt ('my result looks complicated'), there is an error in the problem sheet like a missing factor -1, 2, π, i . . .

Problem 1.1 – Typical electric fields and other numbers (8 points)

(i) Translate the power of a laser pointer into its electric field. Find out typical numbers for a laser pointer and for high-power lasers like those of the *National Ignition Facility* (USA). [Bonus question: Why does the *NIF* work with huge beam diameters?]

(ii) Argue that for a 'typical molecule' (not too large), the electric dipole moment is of the order of $d \sim e a_0$ where e is the electron charge and a_0 the Bohr radius.

'Argue' means: write down hand-waving arguments.

(iii) The interaction energy of an electric dipole $\mathbf{d}$ in a field $\mathbf{E}$ is given by $V_{\text{int}} = -\mathbf{d} \cdot \mathbf{E}$. Estimate the laser power where this energy is comparable to the photon energy $\hbar\omega$, for a 'typical' atom or molecule.

(iv) Experiments in ultrafast optics here in Potsdam work with light pulses whose energy (per pulse) is in the $1\,\mathrm{mJ}$ range, with a spectrum that peaks in the visible.

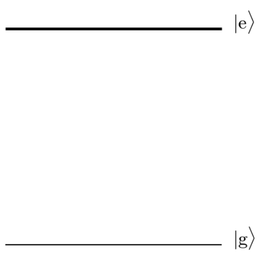
Check out details with the groups of M. Bargheer and M. Gühr: find where their rooms are and ask somebody from the group, or check out their web pages.

How many photons does such a pulse contain? Imagine that the pulse irradiates a metallic surface and is absorbed there: how deep is the penetration depth of the light in the metal? Imagine that the light pulse is really tightly focused and gets absorbed in a volume of $(100\,\mathrm{nm})^3$. If the atoms in this volume redistribute the absorbed energy among them (typically by electron-electron collisions), how big is the increase in energy per atom? What increase of temperature does this represent?

Problem 1.2 – Spectral lines (4 points)

Around 1820, Joseph Fraunhofer discovered the spectral lines of the spectrum of the Sun and identified a doublet (pair of lines) that he had seen before in laboratory experiments. Find out which wavelength, frequency, photon energy have these D lines of sodium. How large is their "natural linewidth"? Which quantum numbers can you assign to the two (or three?) states in the sodium atom that are involved in the quantum jumps that generate the D-line emission. How large is the "natural life time" of the two (three?) states?

Problem 1.3 – Quantum dynamics (8 points)



The sketch on the left is often used for two-level systems, or 'q-bits' (qubits, qbits) in modern jargon. They can be implemented with different physical systems, leading to a variety of notations. (i) If α, β are the probability amplitudes of the two states, write in Dirac bra-ket notation a general superposition state. (ii)

If the levels are electronic states of an effective 'one-electron system' like an alkali atom, write down the electronic wave function ψ for a generic superposition state. Specialise to the hydrogen atom with the energy levels $|\mathrm{g}\rangle = |1\mathrm{s}\rangle$ and $|\mathrm{e}\rangle = |2\mathrm{p}_z\rangle$ and look up the formulas for the hydrogen wave functions. Compute for the superposition state given above the mean value of the z-coordinate of the electron: $\langle z \rangle = \langle \psi | z | \psi \rangle$. Show that for $\alpha = 0$ or $\beta = 0$, one gets $\langle z \rangle = 0$. [10 Bonus Points]

(iii) From the solution to the time-dependent Schrödinger equation, we know the time dependence of the amplitudes α and β: $\alpha(t) = \alpha\, \mathrm{e}^{-\mathrm{i}E_{1\mathrm{s}}t/\hbar}$, and analogously for β and $E_{2\mathrm{p}} = E_{1\mathrm{s}} + \hbar\omega$. From this, you can build the state $|\psi(t)\rangle$. Show that $\langle z(t) \rangle$ oscillates at the Bohr frequency $\omega/2\pi$ ("Lyman-α line"). How big should be the power P radiated by the electron according to classical electrodynamics? If you compute according to the rule $P = \hbar\omega\gamma$ a rate γ, you get a quite accurate estimate for the "natural life time" $1/\gamma$ of the state $|2\mathrm{p}_z\rangle$: is this in the order of femto-seconds, nano-seconds, years?