Einführung in die Quantenoptik I

Wintersemester 2019/20
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Problem Set #4 Hand out: 19 Nov 2019
Hand in: 03 Dec 2019

Problem 4.1 – Dephasing, lifetime, decay and so on (5 points)

In the context of spin resonance or two-level systems in general, one introduces two different time scales, called T_1 and T_2. One time (longer) is relevant for the lifetime of the excited state, while the second time (shorter) governs the 'coherence' of the system and depends on mechanisms like 'dephasing'. Other people talk of 'free induction decay' for the shorter time. Try to find estimates for numbers in typical systems, having in mind atoms in the gas phase and emitters embedded in a solid matrix like quantum dots or impurity spins. Is there an upper limit like $T_2 \leq T_1$ (or $T_1 \leq 2T_2$)? What do people mean by T_2^* or T_1^*? Open a contest to find a system with a very long coherence time.

Problem 4.2 – Bloch equations, dipole quadratures, inversion, and absorption (10 points)

One possible way of writing the optical Bloch equations is the following

$$\begin{aligned}
\frac{\mathrm{d}s_1}{\mathrm{d}t} &= \Delta s_2 - \Gamma s_1 \\
\frac{\mathrm{d}s_2}{\mathrm{d}t} &= -\Delta s_1 - \Omega s_3 - \Gamma s_2 \\
\frac{\mathrm{d}s_3}{\mathrm{d}t} &= \Omega s_2 - \gamma(s_3 + 1)
\end{aligned} \tag{4.1}$$

where $s_1 \ldots s_3$ are the components of the Bloch vector, $\Delta = \omega_L - \omega_A$ is the detuning, and Ω the (real-valued) Rabi frequency (taken here as a constant). These equations are written in the *rotating frame* (in the dipole, a carrier wave $\sim \mathrm{e}^{-\mathrm{i}\omega_L t}$ was factored off).

(i) Show that one gets these equations from the two ingredients introduced in the lecture: the effective Hamiltonian $H_{\mathrm{AL}} = -\frac{1}{2}\hbar\Delta\sigma_3 + \frac{1}{2}\hbar\Omega\sigma_1$ and the so-called Lindblad form for the time evolution of operator averages

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle A\rangle = \frac{\mathrm{i}}{\hbar}\langle[H_{\mathrm{AL}}, A]\rangle + \frac{1}{2}\sum_j\langle[L_j^\dagger, A]L_j\rangle + \frac{1}{2}\sum_j\langle L_j^\dagger[A, L_j]\rangle \tag{4.2}$$

with two so-called jump operators $L_1 = \sqrt{\gamma}\,\sigma = \frac{1}{2}\sqrt{\gamma}(\sigma_1 - \mathrm{i}\sigma_2)$ and $L_2 = \sqrt{\Gamma'}\,\sigma_3$. Fix the quantity Γ' by working out the equation of motion for $A = \sigma_1$.

(ii) Combine the first two equations into one for the complex dipole amplitude $s_1 - \mathrm{i}s_2$. Switch to the notation $P = s_1 - \mathrm{i}s_2$, $E = -\Omega$, $N = \sigma_3$ (inspired by "polarisation", "electric field", and "inversion") and show that the inversion increases when the product of E and the imaginary part of P is positive – this explains why this quadrature of P is responsible for absorption.

(iii) Coming back to Eq.(4.1), compute the stationary solutions for s_2 and s_3 and plot them as a function of the detuning Δ. The plot of s_2 gives the *absorption spectrum*, while s_3 gives the *excitation spectrum*. Find an estimate for the widths of these spectra (the widths will in general depend on the Rabi frequency).

Problem 4.3 – Photons (4 points)

Search for answers to the question "What is a photon?". Which experimental facts are taken as proofs that light particles, called photons exist?

Try to find an "explanation" of the photoelectric effect that does not involve photons, but only electromagnetic waves and not even electrons as particles. (See Lamb and Scully at https://www.ntrs.nasa.gov/search.jsp?R=19690054849)