Bloch vector & Rabi oscillations

Blatt #3
Python
Bloch vector

on resonance (laser $\omega_L = \omega_A$ atom)

real vector
3 components
$s_{1,2,3}$ or u, v, w

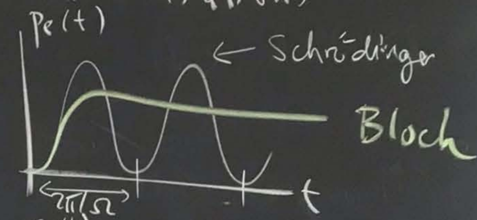
$c_e(t) = i \sin(\Omega t/2)$

$c_g(t) = \cos(\Omega t/2)$

Rabi freq $\frac{\Omega}{2} = -\vec{d}^*_{ge} \cdot \vec{E}_L \sim \sqrt{I_L}$

rotating-wave = resonance (approx'n)

Rabi oscillations



$s_3 = \langle \sigma_3 \rangle = \langle |e\rangle\langle e| - |g\rangle\langle g| \rangle = |c_e|^2 - |c_g|^2$ = $-\cos \Omega t$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \leftrightarrow |e\rangle$ "Inversion"

$s_1 + i s_2 = 2 c_e^* c_g$ (oder $2 c_e c_g^*$)

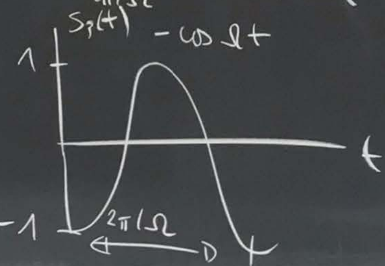
$= \langle \sigma_1 + i\sigma_2 \rangle$

$\overset{?}{=} 2 \langle |e\rangle\langle g| \rangle$

$= 2\, c_e^* c_g$ ← sensitive to superpositions

e.g. $2 c_e^* c_g = -\, i \sin(\Omega t)$

Projection of rotating vector $\vec{s}(t)$



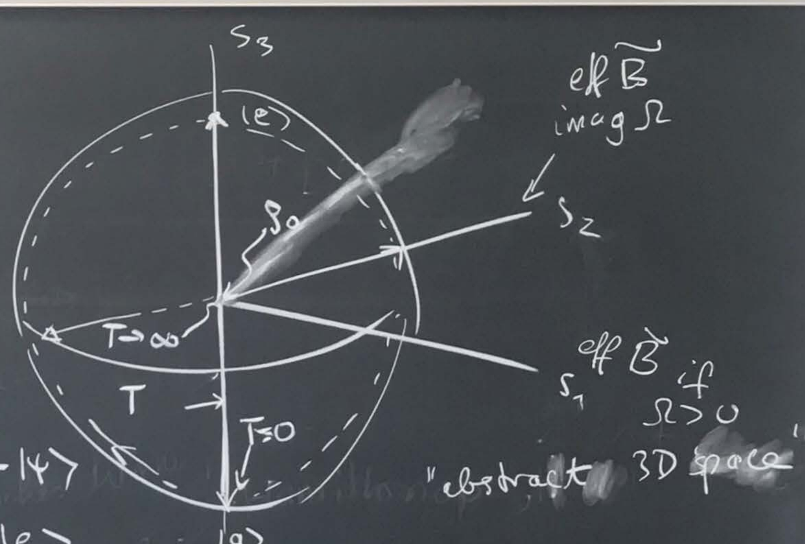
Rabi oscillation

$s_3(t) = -\cos \Omega t$

$s_2(t) = -\sin \Omega t$

$s_1(t) = 0$

rotation ("precession") around s_1-axis ("effective magn. field")



2π pulse (time $\frac{2\pi}{\Omega}$) $|\psi\rangle \longmapsto -|\psi\rangle$

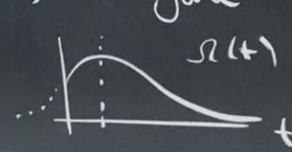
π pulse (π/Ω) $|g\rangle \longmapsto i|e\rangle$ (Rabi-flip)

$\pi/2$ pulse ($\pi/2\Omega$) $|g\rangle \longmapsto \frac{|g\rangle + i|e\rangle}{\sqrt{2}}$ (same weight for $|g\rangle$ & $|e\rangle$)

Hadamard gate

Phase is fixed by complex Ω

adjust Rabi freq Ω, intera. time t



pure states = sphere (surface) of Bloch "ball"

$\uparrow$

"wave functions" $|\psi\rangle = c_e|e\rangle + c_g|g\rangle$ $\leftarrow$ 3 free parameters for kets

$$\vec{s}^2 = s_3^2 + |s_1 + i s_2|^2 = (|c_e|^2 - |c_g|^2)^2 + 4|c_e^* c_g|^2 = (|c_e|^2 + |c_g|^2)^2 = 1$$

$$4|c_e|^2|c_g|^2$$

(2 free parameters for pure states as Blochvectors)

outside – "forbidden" (pf #3.2) if $|\vec{s}|>1$, then probability $\frac{1-|\vec{s}|}{2} < 0$

eigenvalue of mixed state (density matrix)

inner sphere – "mixed states"

generalises QM + class. statistics

density operator (matrix)

/ part of bigger system

– incomplete knowledge on preparation

average $p_1\langle\psi_1|A|\psi_1\rangle + p_2\langle\psi_2|A|\psi_2\rangle$

$= \mathrm{tr}\left[\left(p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2|\right)A\right]$

$|\psi\rangle = \begin{cases} |g_1\rangle & \text{w/ } 99\% = p_1 \\ |e_1\rangle & \text{w/ } 1\% \text{ "error" } p_2 \end{cases}$

$\sqrt{0.99}\,\cancel{|g\rangle} + 0.1|e\rangle$

$\downarrow$ density matrix

$$\varrho = 99\%\,|g\rangle\langle g| + 1\%\,|e\rangle\langle e| = \begin{pmatrix} 1\% & 0 \\ 0 & 99\% \end{pmatrix}$$

$$= p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2| + \dots$$

general basis for ϱ: eigenvectors will differ from incomplete knowledge preparation $\leftarrow$ need not be orthogonal (!)

example

$\vec{s} = 0$

$\sqrt{}$ ex #3.2

$$\varrho_0 = \frac{\mathbb{1}}{2} + \frac{\vec{s}\cdot\vec{\sigma}}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{cases} |e\rangle & 50\% \\ |g\rangle & 50\% \end{cases}$$

"completely mixed state"

$$= \begin{cases} \frac{|g\rangle + i|e\rangle}{\sqrt{2}} & 50\% \\ \frac{|g\rangle - i|e\rangle}{\sqrt{2}} & 50\% \end{cases}$$

"thermal state"

$$\varrho_T = \frac{e^{-\hbar\omega_A/kT}|e\rangle\langle e| + |g\rangle\langle g|}{Z}$$

Z = partition function $= 1 + e^{-\hbar\omega_A/kT}$

$\hookrightarrow \mathrm{tr}(\varrho_T) = 1$

$s_{1,2} = 0 \quad s_3 = -\tanh\left(\frac{\hbar\omega_A}{2kT}\right)$